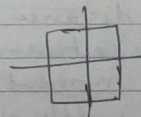


① Max. principal stress theory (Rankine)

$$\sigma = \frac{S_{\text{int}}}{f_{\text{as}}}$$

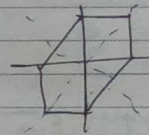


ordinary cast iron &
brittle materials

② max. shear stress theory

$$C = \frac{0}{2}$$

(Quest/Tresca)
safe results for
ductile



③ ^{ductile} strain energy theory (Haigh) (\approx ductile)

$$\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - \frac{2}{n}(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1) = \sigma_c^2$$

④ shear strain energy theory (vaise-Henry)

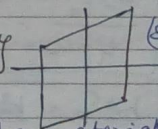
$$(a_1 - a_2)^2 + (a_2 - a_3)^2 + (a_3 - a_1)^2 = 2a_c^2$$

best for ductile

⑤ max. principal strain theory

$$\sigma_1 - \frac{1}{m}(\sigma_2 + \sigma_3) < \sigma_e$$

brittle material



→ Strain energies

$$\text{tension} = \sqrt{\frac{2}{2\epsilon}} \times V$$

$$\text{Shear} = \left| \frac{\tau^2}{2c} \times V \right|$$

torsion = $\frac{T^2}{4C} \times V$

$$\frac{Z^2}{4c} \left(\frac{R_{\infty}^2}{r^2} \right) \psi$$

→ Shearing Stresses

$$\tau = \frac{SA\bar{y}}{Ib}$$

S = shear force

A = cross-sectional area

\bar{y} = distance from neutral section

I = moment of inertia

b = width of beam at point where shear force is considered.

Rectangular

$$\tau_{max} = \frac{3}{2} \tau_{mean}$$

Solid circular

$$\tau_{max} = \frac{4}{3} \tau_{mean}$$

→ Transient Conduction

$$\frac{t - t_a}{t_i - t_a} = e^{-\frac{hAC}{\rho VC} \tau}$$

$$\frac{hAC}{\rho VC} = \frac{hL_c}{K} \left(\frac{\alpha \tau}{L_c^2} \right) = Bi \cdot F$$

$$Q = -hA(t_i - t_a) e^{-Bi \cdot F} \quad \text{— instantaneous heat transfer}$$

$$Q' = \rho VC(t_i - t_a) (e^{-Bi \cdot F} - 1) \quad \text{— total heat transfer}$$

$$\text{Time constant} = \frac{\rho VC}{hA}$$

Response

time taken for temp. change to reach 36.8% of its final value.

Sensitivity — time taken for temp. change to reach 36.8% of initial value

→ Radiation

$$\text{Wien's disp. law } (E_b)_{max} = 1.285 \times 10^{-5} T^5 \frac{W}{m^2}$$

$$\lambda_m T = 2829$$

→ Radiation Exchange b/w 2 non-black bodies

$$Q_1 = \frac{E_1}{\frac{E_1}{\epsilon_1} + 1 - \epsilon_1}$$

$$Q_2 = \frac{E_2}{\frac{E_2}{\epsilon_2} + 1 - \epsilon_2}$$

$$\text{interchange factor } f_{1-2} = \frac{1}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1}$$

parallel flat plates

$$\text{parallel cylinders } \left(\frac{1}{\frac{1}{\epsilon_1} + \frac{A_1}{A_2} \left(\frac{1}{\epsilon_2} - 1 \right)} \right)$$

$$\text{Effectiveness of fin} = \frac{Q_{\text{with fin}}}{Q_{\text{without fin}}}$$

→ Complex Numbers

$$u + iv = \text{Real} + i(\text{Imaginary})$$

$$\text{De Moivre's Theorem } (\cos + i \sin \theta)^n = \cos n\theta + i \sin n\theta$$

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots = \sum_{i=0}^{\infty} \frac{x^i}{i!}$$

Hyperbolic functions

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

$$\tanh x = \frac{\sinh x}{\cosh x}$$

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$$\sin(ix) = i \sinh x$$

$$\sinh x = i \sin x$$

$$\cos(ix) = \cosh x$$

$$\cosh x = \cos x$$

$$\tan(ix) = i \tanh x$$

$$\tanh x = i \tan x$$

CR - conditions

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$$

$$\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

Cauchy's Theorem

If $f(z)$ is analytic in closed curve C ,

$$\int_C f(z) dz = 0.$$

Cauchy's Integral Formula

$f(z)$ is analytic in closed curve C except at point a .

$$f(a) = \frac{1}{2\pi i} \int \frac{f(z) dz}{z-a}$$

$$f^{(n)}(a) = \frac{n!}{2\pi i} \int \frac{f(z) dz}{(z-a)^{n+1}}$$

Residue

~~The~~ coeff. of $\frac{1}{z-a}$ in expansion of $f(z)$ around isolated singularity is called residue

$$\text{Res}(f, a) = \frac{1}{2\pi i} \int_C f(z) dz$$

$$\begin{aligned} \int_C f(z) dz &= \int_{C_1} f(z) dz + \int_{C_2} f(z) dz + \dots + \int_{C_n} f(z) dz \\ &= 2\pi i [\text{Res}(f, a_1) + \text{Res}(f, a_2) + \dots + \text{Res}(f, a_n)] \end{aligned}$$

$$\text{Res } f(a) = \lim_{z \rightarrow a} [(z-a)f(z)] \rightarrow \text{pole of order 1}$$

$$\text{Res } f(a) = \frac{1}{(n-1)!} \left[\frac{d^{n-1}}{dz^{n-1}} [(z-a)^n f(z)] \right]_{z=a}$$

Matrices

Solⁿ of eqns. Homogeneous Equations

r = rank of coeff. matrix.

n = no. of coefficients.

$$\Rightarrow r=n, \quad x_i=0$$

$\Rightarrow r < n$ only many solutions

→ Series expansions

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots$$

$$\log(x+1) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$$

$$\text{Maclaurin series} = f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0) + \dots$$

$$\text{Taylor Series} = f(x+a) = f(a) + \frac{x}{1!} f'(a) + \frac{x^2}{2!} f''(a) + \frac{x^3}{3!} f'''(a) + \dots$$

→ Bearings

$$C_0 = \text{static load carrying capacity} = \frac{K d^2 z}{5}$$

$$P = \text{dynamic load} = X V F_r + Y F_a$$

$$L_{10} = \text{life in million revolutions} = \left(\frac{C}{P} \right)^p$$

$$L_{10} = \left(\frac{C}{P} \right)^p = \frac{GOL_{10} n}{10^6}$$

$p = 3$ (ball bearings)

$p = \frac{10}{3}$ (roller bearings)

$$\text{Sommerfeld No.} = \frac{\mu n d^3}{P c^2}$$

μ = viscosity, n = rpm

P = unit bearing pressure

d = dia of journal (inner circle)

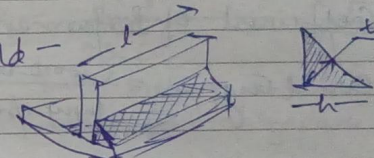
c = clearance $(D-d)$

$$P_c = \sqrt{\frac{\sum N_i P_i^2}{\sum N_i}}$$

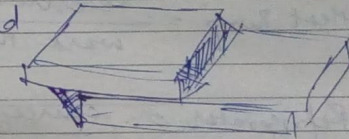
$$\frac{L}{L_0} = \left[\frac{\ln(VR)}{\ln(VR_0)} \right]^{1/17}$$

→ Welding

Parallel Fillet weld -
 $P = 0.407 \text{ kJ/cm}$



Transverse fillet weld
 $P = 0.707 \text{ kJ/cm}$



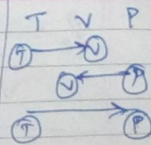
→ Work done

Process	Q	W	ΔS
$PV = C$	$PV_1 \ln \left(\frac{V_2}{V_1} \right)$	$PV_1 \ln \left(\frac{V_2}{V_1} \right)$	$R \ln \left(\frac{V_2}{V_1} \right)$
$PV^n = C$	0	$\frac{P_1 V_1 - P_2 V_2}{n-1}$	0
$P/T = C$	$G \Delta T$	0	$G \ln \frac{T_2}{T_1}$
$N/T = C$	$G \Delta T$	$P \Delta V$	$G \ln \left(\frac{T_2}{T_1} \right)$
$PV^n = C$	$\frac{G(n-1) \Delta T}{1-n}$	$\frac{P_1 V_1 - P_2 V_2}{n-1}$	$G \left(\frac{n-1}{n-1} \right) \ln \left(\frac{T_2}{T_1} \right)$

$$\Delta S = C_v \ln T_2/T_1 + R \ln V_2/V_1$$

$$= C_v \ln P_2/P_1 + C_v \ln V_2/V_1$$

$$= C_p \ln T_2/T_1 - R \ln P_2/P_1$$



→ Coefficient of Performance

Carnot Engine = $\frac{\text{work output}}{\text{input from reservoir}}$

Heat Pump = $\frac{\text{reservoir heat}}{\text{work input}}$

Refrigerator = $\frac{\text{source heat}}{\text{work input}}$

→ Clutches

uniform pressure theory

$$R = \frac{2}{3} \left(\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2} \right)$$

$$T = \mu W R$$

$$\text{pressure} = \frac{W}{\pi(r_1 - r_2)}$$

uniform wear theory

$$R = \frac{r_1 + r_2}{2}$$

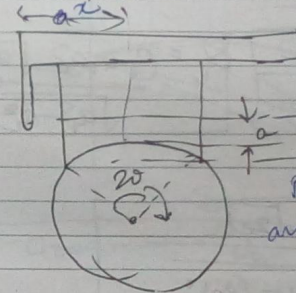
$$T = \mu W R$$

$$\text{pressure} = \frac{W}{\pi(r_1^2 - r_2^2)}$$

cone clutch $T = \frac{\mu W R}{\sin \alpha}$

→ Brakes

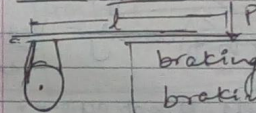
Shoe brake



clockwise
braking torque $T_B = \frac{\mu P r_1}{\pi - \mu a}$

Initial braking power = $T_B \times \omega$
average braking power = $\frac{0 + T_B \times \omega}{2}$

Band Brake



braking force = $T_1 - T_2$
braking torque = $(T_1 - T_2) R$

Band & block brake

$$T_1 = \frac{(1 + \mu \tan \theta)^n}{T_2 (1 - \mu \tan \theta)^n}$$

→ Projectile motions

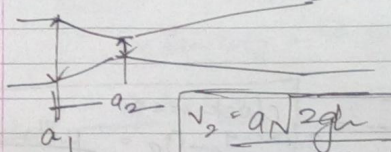
highest point = $\frac{u^2 \sin^2 \alpha}{2g}$

Time of flight = $\frac{2u \sin \alpha}{g}$

Time taken to reach the highest point = $\frac{u \sin \alpha}{g}$

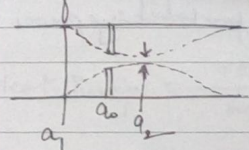
Range = $\frac{u^2 \sin 2\alpha}{g}$

Venturimeter



$$V_2 = \frac{a_1 \sqrt{2gh}}{\sqrt{a_1^2 - a_2^2}} \quad Q = \frac{C_d a_1 a_2 \sqrt{2gh}}{\sqrt{a_1^2 - a_2^2}}$$

orificemeter



$$V_2 = \frac{\sqrt{2gh}}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2 C_c^2}}$$

$$Q = \frac{C_d a_0 a_1 \sqrt{2gh}}{\sqrt{a_1^2 - a_0^2}}$$

coeff of velocity = $\frac{V_{actual}}{V_{ideal}}$

Coeff. of contraction = $\frac{a_2}{a_0}$

Design

Fluctuating loads

stress concentration $K_t = \frac{\sigma_{max}}{\sigma_0} = \frac{\frac{S_{ut}}{FOS}}{P/A}$

Endurance limit = max. stress of a specimen can bear without failure for ∞ no. of cycles

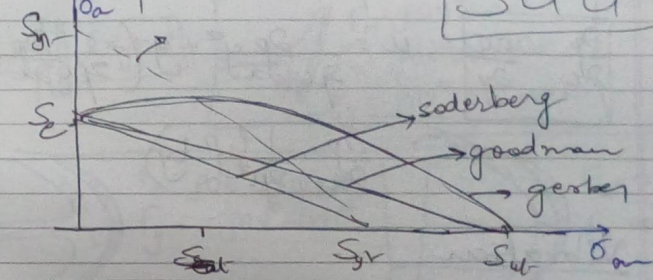
Notch sensitivity $K_f = \frac{E.L. \text{ of notched specimen}}{E.L. \text{ of notch free specimen}}$

$$K_f = 1 + q(K_t - 1)$$

$$S_e = K_a K_b K_c K_d S_e'$$

\downarrow surface finish \downarrow size effect \downarrow reliability \downarrow endurance limit of notch free specimen

Finite life



SAG

Goodman $\frac{\sigma_m}{S_{ur}} + \frac{\sigma_a}{S_e} = 1$

Gerber $\left(\frac{S_m}{S_{ur}}\right)^2 + \frac{S_a}{S_e} = 1$

Viscous Flow

Flow through circular pipe -

$$u = \left(\frac{-2p}{2\mu} \right) \left(\frac{r}{2}\right) \quad u = -\frac{1}{4\mu} \frac{dp}{dx} [r^2 - r_0^2]$$

$$u = \frac{R^2}{8\mu} \left(\frac{-dp}{dx}\right) \quad \frac{u_{max}}{u} = 2$$

Pressure drop for given length

$$P_1 - P_2 = \Delta p = \frac{32\mu u L}{D^2}$$

$$\text{loss of pr. head} = \frac{P_1 - P_2}{\rho g} = \frac{32\mu u L}{\rho g D^2}$$

Flow Between 2 parallel plates

$$\frac{\partial p}{\partial x} = \frac{\partial \tau}{\partial y} \quad u = \frac{1}{2\mu} \frac{\partial p}{\partial x} y^2 + y \left(-\frac{1}{2\mu} \frac{\partial p}{\partial x} \right)$$

$$= \frac{\partial p}{\partial x} \cdot \frac{1}{2\mu} (y^2 - y)$$

→ Vibration

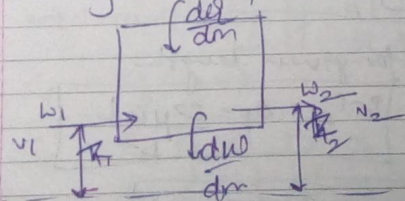
speed = frequency × wavelength
frequency × time period = 2π

→ Dimensionless Numbers

$Pr = \frac{\mu C_p}{k}$	$Re = \frac{\rho V L}{\mu}$	$Nu = \frac{h L}{k}$	$Fr = \frac{V}{\sqrt{g L}}$
$St = \frac{Nu}{Re Pr}$	$Gr = \frac{L^3 \rho \beta \Delta T}{\mu^2}$		

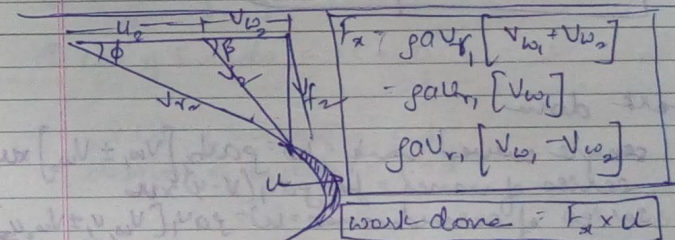
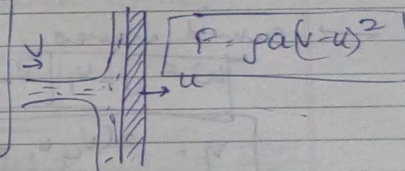
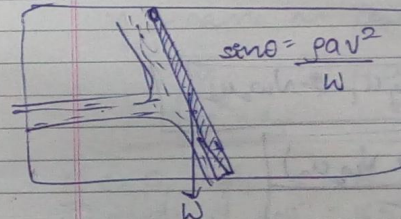
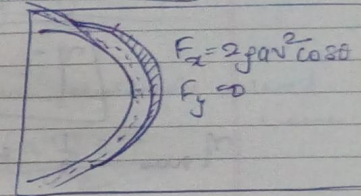
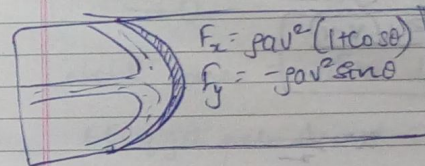
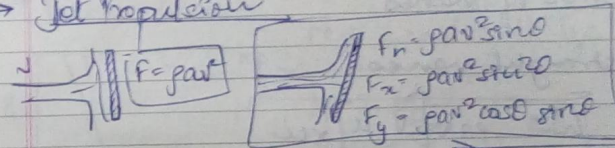
→ SFTC

steady, incompressible, inviscid



$$h_1 + \frac{v_1^2}{2} + z_1 g + \frac{dw}{dm} = h_2 + \frac{v_2^2}{2} + z_2 g + \frac{dw}{dm}$$

→ Jet Propulsion



water enters and leaves without shock

radial input $\Rightarrow v_{r1} = v_{r2}$
radial o/p $= R = \frac{r}{2}$

series of vanes

→ straight vanes

$$W = \rho a V [V - u] x u$$

$$\eta = \frac{2u(V-u)}{V^2}$$

$$\eta_{max} = \frac{1}{2} \text{ at } u = \frac{V}{2}$$

→ curved vanes

$$W = \rho a V_1 [V_{w1} u_1 \pm V_{w2} u_2]$$

$$\eta = \frac{2(V_{w1} u_1 \pm V_{w2} u_2)}{V^2}$$

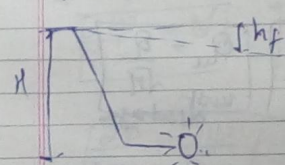
work done

$$\text{single curved vane} = W = \rho a V_1 [V_{w1} \pm V_{w2}] x u$$

$$\text{series of vanes} = W = \rho a V_1 (V - u) x u$$

$$\text{series of curved vanes} = W = \rho a V_1 [V_{w1} u_1 \pm V_{w2} u_2]$$

→ Pelton wheel



$$\text{Net head} = H - h_f$$

$$h_f = \frac{4fLV^2}{2gD}$$

work done = work done for simple curved vane

$$\text{KE of jet per second} = \frac{\rho a V_1 V_1^2}{2}$$

η_h = same as curved vanes

$$\eta_{h_{max}} = \frac{1 + \cos \phi}{2}$$

$$\textcircled{1} V = C_v \sqrt{2gH}$$

$$\textcircled{2} u = \phi \sqrt{2gH} = \frac{\pi DN}{60}$$

$$\textcircled{3} \text{ angle of deflection} = 165^\circ$$

$$\textcircled{4} \text{ No. of buckets} = \frac{15 + D}{2d}$$

$$\text{width of buckets} = 5d$$

$$\text{depth of buckets} = 1.2d$$

→ Radial flow Reaction Turbines

W = series of curved vanes

$$u_1 = \frac{\pi DN}{60}$$

$$u_2 = \frac{\pi D_2 N}{60}$$

$$\eta_h = \frac{V_{w1} u_1 \pm V_{w2} u_2}{gH}$$

radial discharge $V_{w2} = 0$

$$\text{Discharge (Q)} = \pi DBV_f \text{ (vel. of flow at inlet)}$$

dia of runner at inlet

width of runner at inlet

Francis turbine - inward radial flow reaction turbine

→ Specific Quantities

$$N_s = \frac{N \sqrt{P}}{H}$$

with $\frac{H}{H}$

$$N_u = \frac{N}{\sqrt{H}}$$

with speed

$$Q_u = \frac{Q}{\sqrt{H}}$$

with discharge

$$P_u = \frac{P}{H^{3/2}}$$

with power

→ Kinematics

⇒ continuity eqn $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$

⇒ continuity eqn acceleration

$$a_x = \frac{du}{dt} = \underbrace{\frac{\partial u}{\partial x} \frac{dx}{dt} + \frac{\partial u}{\partial y} \frac{dy}{dt} + \frac{\partial u}{\partial z} \frac{dz}{dt}}_{\text{convective accel}} + \underbrace{\frac{\partial u}{\partial t}}_{\text{local accel}}$$

Velocity potential for (ϕ)

$$u = \frac{\partial \phi}{\partial x}$$

$$v = \frac{\partial \phi}{\partial y}$$

$$w = \frac{\partial \phi}{\partial z}$$

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0$$

Streamline & equipotential flow are identical.

Stream Function

$$\frac{\partial \psi}{\partial x} = v$$

$$\frac{\partial \psi}{\partial y} = -u$$

$$\frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y}$$

$$\frac{\partial \phi}{\partial y} = -\frac{\partial \psi}{\partial x}$$

Rotational components

$$\omega_x = \frac{1}{2} \left[\frac{\partial v}{\partial y} - \frac{\partial u}{\partial z} \right]$$

$$\omega_y = \frac{1}{2} \left[\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \right]$$

$$\omega_x = \frac{1}{2} \left[\frac{\partial v}{\partial y} - \frac{\partial u}{\partial z} \right]$$

$$\omega_x = \frac{1}{2} \left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right]$$

$$\begin{aligned} \omega_x &= x \cdot y \cdot z \\ \omega_y &= z \cdot x \cdot y \\ \omega_z &= y \cdot x \cdot z \end{aligned}$$

→ Jacobians

$$J \left(\frac{u, v}{x, y} \right) = \frac{\partial(u, v)}{\partial(x, y)} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix}$$

⇒ $J = \frac{\partial(u, v)}{\partial(x, y)}$, $J^* = \frac{\partial(x, y)}{\partial(u, v)}$ then $J J^* = 1$

⇒ $J \left(\frac{u, v}{x, y} \right) = J \left(\frac{u, v}{\xi, \eta} \right) J \left(\frac{\xi, \eta}{x, y} \right)$

→ Wronskians

$$W(y_1, y_2) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

y_1, y_2 are linearly independent if $W \neq 0$

→ Fourier

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

$$a_0 = \frac{1}{\pi} \int_a^{a+2\pi} f(x) dx$$

$$a_n = \frac{1}{\pi} \int_a^{a+2\pi} f(x) \cos nx dx$$

$$b_n = \frac{1}{\pi} \int_a^{a+2\pi} f(x) \sin nx dx$$

→ Numerical Methods

RK2

input x_0, y_0, x_g, h

$$n = \frac{x_g - x_0}{h}$$

n-steps

$$K_1 = h * f(x_0, y_0)$$

$$K_2 = h * f(x_0 + h, y_0 + K_1)$$

$$K = \frac{K_1 + K_2}{2}$$

$$y_g = y_0 + K$$

$$x_0 = x_0 + h$$

$$y_0 = y_g$$

RK4

input x_0, y_0, x_g, h

$$n = \frac{x_g - x_0}{h}$$

n-steps

$$K_1 = h * f(x_0, y_0)$$

$$K_2 = h * f(x_0 + \frac{h}{2}, y_0 + \frac{K_1}{2})$$

$$K_3 = h * f(x_0 + h, y_0 + K_2)$$

$$K_4 = h * f(x_0 + h, y_0 + K_3)$$

$$K = \frac{K_1 + 2K_2 + 2K_3 + K_4}{6}$$

$$y_0 = y_0 + K$$

$$x_0 = x_0 + h$$

$$y_0 = y_g$$

Governer

Euler

$$y = f(x, y)$$

x_0, x_g, y_0, h

$$n = \frac{x_g - x_0}{h}$$

n-steps

$$y_g = y_0 + h * f(x_0, y_0)$$

$$x_0 = x_0 + h$$

$$y_0 = y_g$$

Improved Euler

Euler Modified

$$y = f(x, y)$$

$x_0, x_g, y_0, \text{accuracy}$

$$n = 2$$

$$h = \frac{x_g - x_0}{2}$$

2-steps

$$y_1 = y_0 + h * f(x_0, y_0)$$

$$y_{11} = y_0 + \frac{h}{2} [f(x_0, y_0) + f(x_0 + h, y_1)]$$

if $|y_{11} - y_1| < \text{accuracy}$

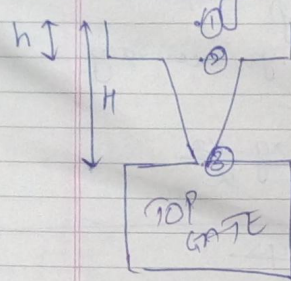
$$y_1 = y_{11}$$

$$y_{11} = y_0 + \frac{h}{2} [f(x_0, y_0) + f(x_0 + h, y_1)]$$

$$x_0 = x_0 + h$$

$$y_0 = y_{11}$$

→ Moulding



Time taken to fill mould

→ top gate $t = \frac{V_{mold}}{A_3 \sqrt{2gh}}$

→ bottom gate $t = \sqrt{\frac{2}{g}} \frac{A_m}{A_g} [\sqrt{H} - \sqrt{H-h}]$

Sprue
 $\frac{A_3}{A_2} = \sqrt{\frac{h}{H}}$

Design of Riser

$t = \text{solidification time} = K \left(\frac{V}{SA} \right)^2$

Freezing Ratio = $\frac{(SA/V)_{casting}}{(SA/V)_{riser}}$ → must be > 1 .

Chene's relationship

$x = 0.1 \rightarrow +1$
 $y = 0.03 \rightarrow \text{for steel}$

Top Riser $(V/A) = d/6$ → same for side riser

→ Rolling

max. redⁿ in thk $= \sqrt{\mu^2 R} = \Delta h_{max}$

roll contact length $= \sqrt{R \Delta h}$

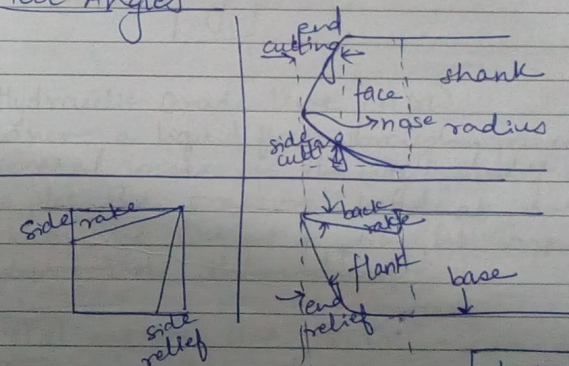
contact angle / angle of bite $\tan \alpha = \sqrt{\frac{\Delta h}{R}}$

Roll force

$P_{av} = 2K \left(1 + \frac{\mu L}{2h_{av}} \right)$

$K = \frac{\sigma_y}{\sqrt{3}}$ $h_{av} = \frac{h_i + h_f}{2}$

→ Tool Angles



For orthogonal cutting
 $\frac{\tan(\text{back rake})}{\tan(\text{side rake})} = \tan(\text{side cutting})$

back rake	α_b
side rake	α_s
end relief	γ_e
side relief	γ_s
end cutting	ϕ_e
side cutting	ϕ_s

→ SPOF Coulomb Damping

left to right
right to left

$$\begin{cases} m\ddot{x} + Kx + \mu N = 0 \\ m\ddot{x} + Kx - \mu N = 0 \end{cases}$$

soln

$$L \rightarrow R \quad x(t) = A_1 \cos \omega t + A_2 \sin \omega t - \frac{\mu N}{K}$$

$$R \rightarrow L \quad x(t) = A_3 \cos \omega t + A_4 \sin \omega t + \frac{\mu N}{K}$$

loss of amplitude in every cycle

$$x_n = x_{n-1} - \frac{4\mu N}{K}$$

→ Forced vibration

$$F(t) = F_0 \cos(\omega t)$$

$$\text{w/f or } \textcircled{A} \quad x(t) = x_h(t) + x_p(t)$$

$x_h(t)$ = soln of homogeneous eqn
 $A \cos(\omega t) + B \sin(\omega t)$

$$x_p(t) = C \cos(\omega t)$$

$$C = \frac{F_0}{K - m\omega^2} = \frac{F_0/K}{(1-r^2)}$$

r = frequency
ratio = $\frac{\omega}{\omega_n}$

$$x(t) = A \cos \omega_n t + B \sin \omega_n t + \frac{F_0/K}{(1-r^2)} \cos \omega t$$

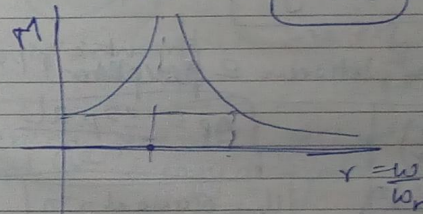
$$\text{w/f or } \textcircled{B} \quad x_p(t) = D \sin \omega t$$

$$D = \frac{F_0}{2m\omega_n}$$

$$x(t) = A \cos \omega_n t + B \sin \omega_n t + \frac{F_0/K}{(1-r^2)} \cos \omega t$$

$$M = \text{magnification factor} = \frac{1}{|1-r^2|}$$

$$= \frac{x_{\text{dynamic}}}{x_{\text{static}}} = \frac{\frac{F_0/K}{(1-r^2)}}{\frac{F_0/K}}$$



Harmonically Forced viscous damped

$$\begin{cases} m\ddot{x} + c\dot{x} + Kx = F(t) \\ x = X \cos(\omega t - \phi) \end{cases}$$

$$x(t) = \left(x_0 - \frac{F_0}{K - m\omega^2} \right) \cos \omega_n t + \left(\frac{x_0}{\omega_n} \right) \sin \omega_n t + \frac{F_0 \cos \omega t}{K - m\omega^2}$$

$$x(t) = x_0 \cos \omega_n t + \frac{x_0}{\omega_n} \sin \omega_n t + \frac{F_0}{K} \frac{\cos \omega t}{(1-r^2)}$$

Harmonically Forced Viscous damped vibration

$$m\ddot{x} + c\dot{x} + Kx = F(t)$$

let $x = X \cos(\omega t - \phi)$ → differentiate \dot{x} & \ddot{x} & substitute in EOM.

compare sines & cosines

$$\phi = \tan^{-1} \left(\frac{2\zeta \omega_n \omega}{\omega_n^2 - \omega^2} \right) = \left(\frac{2\zeta r}{1-r^2} \right)$$

→ Buckling $P_c = \frac{\pi^2 EI}{l_c^2}$ $\frac{d^2 y}{dx^2} + \frac{P_y}{EI} = 0$

i) Both ends hinged $l_c = l$
 ii) one fixed, other free $l_c = 2l$
 iii) one fixed, other pin jointed $l_c = \frac{l}{\sqrt{2}}$

iv) Both fixed $l_c = \frac{l}{2}$

Rankine wheel $\frac{1}{P_{\text{Rankine}}} = \frac{1}{P_{\text{Coulth}}} + \frac{1}{P_{\text{Coulth}}}$

→ Spur Gear

pitch = $\frac{\pi d'}{z}$ → PCD
 z → no. of teeth

diametral pitch $P = \frac{z}{d'}$

module $\frac{d'}{z} = m$

Gear ratio $i = \sqrt{i'}$

$i' = \frac{\text{no. of teeth in gear}}{\text{no. of teeth in LAST gear of final stage}}$

contact ratio = $\frac{\text{length of arc of contact}}{\pi m}$

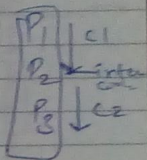
$G = \frac{T}{E}$

①

center distance = $\frac{m(Z_p + Z_g)}{2}$

→ Compressor

$W = \frac{n}{n-1} P_1 V_1 \left[\left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} - 1 \right]$



intercooler pressure = $\sqrt{P_1 P_2}$

→

$dU = m C_v dT$ $dH = m C_p dT$

→

mass of flywheel = $m = 2 \pi R \times A \times \rho$

→

$Q = U + P dV$ holds good for a any process undergone by a closed system.
 $T dS = U + P dV$

→

Gas Turbine

On installing a regenerator, specific output remains same though efficiency is improved.

→

Availability

$W_{\text{min}} = m C_p \left(\frac{T_1^2}{T_f} + T_f - 2 T_1 \right)$

→ Laplace Identities

$$L[e^{at} f(t)] = f(s-a)$$

$$L[L^n f(t)] = (-1)^n \frac{d^n}{ds^n} f(s)$$

$$L[f(at)] = \frac{1}{a} f\left(\frac{s}{a}\right)$$

$$L\left[\frac{f(t)}{t}\right] = \int_s^\infty f(s) ds$$

$$L\left[\int_0^t f(t) du\right] = \frac{f(s)}{s}$$

$$L[f^{(n)}(t)] = s^n f(s) - s^{n-1} f(0) - s^{n-2} f'(0) - \dots - f^{(n-1)}(0)$$

→ Boundary layer

Boundary layer thickness = 0.330

Displacement thk - $\delta^* = \int_0^\delta \left(1 - \frac{u}{U}\right) dy$

Momentum thickness

$$\theta = \int_0^\delta \frac{u}{U} \left(1 - \frac{u}{U}\right) dy$$

Energy thickness

$$\delta_E = \int_0^\delta \frac{u}{U} \left(1 - \frac{u^2}{U^2}\right) dy$$

→ Slenderness Ratio = $\frac{\text{min unsupported length of column}}{\text{min radius of gyration of c-s-area}}$

Buckling factor = $\frac{\text{equivalent length}}{\text{min radius of gyration}}$

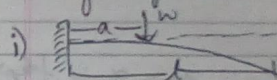
short (8d) med. (30d) long

→ Mobility of A Mechanism

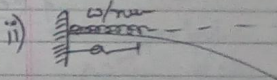
It is the no. of ~~to~~ areas an object can rotate/translate in a 3D space.

$$M = 6S \frac{d^3 y}{dx^2}$$

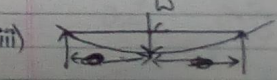
→ Deflⁿ of Beams -



$$y_{\max} = -\frac{Wa^2}{6EI} (3l-a)$$

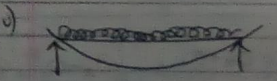


$$y_{\max} = -\left[\frac{Wa^3}{6EI} + \frac{Wa^2}{6EI} (l-a)\right]$$

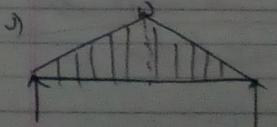


$$y_{\max} = -\frac{Wl^3}{48EI}$$

$$-\frac{wl^3}{48EI}$$



$$y_{\max} = -\frac{5Wl^3}{384EI}$$



$$y_{\max} = -\frac{Wl^4}{120EI}$$

→ $y = A + Bx + Cx^2$

$$\begin{aligned}\sum y &= nA + B\sum x + C\sum x^2 \\ \sum xy &= A\sum x + B\sum x^2 + C\sum x^3 \\ \sum x^2y &= A\sum x^2 + B\sum x^3 + C\sum x^4\end{aligned}$$

→ Multiple regression
 $y = a_0 + a_1x_1 + a_2x_2$

$$\begin{aligned}\sum y &= nA + B\sum x_1 + C\sum x_2 \\ \sum x_1y &= A\sum x_1 + B\sum x_1^2 + C\sum x_1x_2 \\ \sum x_2y &= A\sum x_2 + B\sum x_1x_2 + C\sum x_2^2\end{aligned}$$

→ Newton Raphson
given $y = f(x)$ - solve it
initial guess x_0 .

x_A	x_B	$(x_A - x_B)$
x_0	x_1	
x_1	x_2	
x_2	x_3	
x_3	x_4	

$$x_B = x_A - \frac{f(x_A)}{f'(x_A)}$$

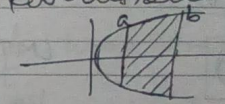
→ 2-D stress system

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2}$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

→ Volume of Revolution

$$y = \sqrt{x}$$



$$V = \int_a^b \pi y^2 dx \quad r=y = \pi \int_a^b y^2 dx$$

$$= \pi \int_a^b x dx$$

→ Grashof's criteria -

4-bar link will contain a rotating pair if sum of shortest & longest link is greater than sum of other two.

→ Mobility of a statically indeterminate system $m \leq -1$.

→ Vorticity vector $= \nabla \times \vec{V}$

→ Hyperbolic PDE

$$A \frac{\partial^2 u}{\partial x^2} + B \frac{\partial^2 u}{\partial x \partial y} + C \frac{\partial^2 u}{\partial y^2} + F\left[x, y, u, \frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}\right] = 0$$

→ MOI of circle $= \frac{\pi d^4}{64}$

J for solid shaft $= \frac{\pi (D^4)}{32}$

J for hollow shaft $= \frac{\pi (D_1^4 - D_2^4)}{32}$

→ Heat Exchangers

$$\dot{Q} = UA(LMTD)$$

$$NTU = \frac{UA}{C_{min}}$$

$$C = \frac{\dot{m} \times \text{sp. heat}}{n}$$

$$LMTD = \frac{\theta_1 - \theta_2}{\ln \frac{\theta_1}{\theta_2}}$$

→ Diesel Engine

$$\eta = 1 - \frac{\text{Heat rejected}}{\text{Heat added}} = 1 - \frac{Q_2}{Q_1}$$

→ Clausius-Clapeyron

$$\frac{dP}{dT} = \frac{L}{T \Delta V} = \frac{h_{fg}}{T v_{fg}} = \frac{s_g - s_f}{v_g - v_f}$$

→ Pitot Tube

$$V = \sqrt{\frac{2 \Delta P}{\rho}} = \sqrt{\frac{2 (P_0 - P) g h}{\rho}}$$

→ If \vec{P} is irrotational, ϕ exists such that $\vec{P} = \text{grad } \phi$ where ϕ is called scalar potential ϕ .

$$\text{grad } \phi = \frac{\partial \phi}{\partial x} \hat{i} + \frac{\partial \phi}{\partial y} \hat{j} + \frac{\partial \phi}{\partial z} \hat{k}$$

→ For cylinder $\sigma_1, \sigma_2, \sigma_3$

$$\sigma_1 = \frac{p d}{4t} - \text{hoop stress}$$

$$\sigma_2 = \frac{p d}{4t} = \text{longitudinal stress}$$

→ Soderberg relation

$$\sigma_1 = \text{given} \quad \sigma_2 = \text{given}$$

$$\bar{\sigma}_y = 0.55 \sigma_u$$

$$\bar{\sigma}_e = 0.5 \sigma_u$$

$$\bar{\sigma}_m = \frac{\sigma_1 + \sigma_2}{2}$$

$$\bar{\sigma}_v = \frac{\sigma_1 - \sigma_2}{2}$$

$$\frac{\bar{\sigma}_m}{\sigma_y} + \frac{\bar{\sigma}_v}{\sigma_e} = \frac{1}{\text{fos}}$$

→ Fins

① cylindrical

$$\frac{t - t_a}{t_0 - t_a} = e^{-mx}$$

$$Q_{fin} = \sqrt{h P K A} (t_0 - t_a)$$

②

$$\frac{t - t_a}{t_0 - t_a} = \frac{\cosh m(L-x)}{\cosh mL}$$

$$Q = \sqrt{h P K A} (t_0 - t_a) \tanh mL$$

③

$$\frac{t - t_a}{t_0 - t_a} = \frac{\cosh m(L-x) + \frac{h}{km} \sinh m(L-x)}{\cosh mL + \frac{h}{km} \sinh mL}$$

$$Q_{fin} = \sqrt{h P K A} (t_0 - t_a) \left[\frac{\tanh mL + \frac{h}{km}}{1 + \frac{h}{km} \tanh mL} \right]$$

→ Numerical Methods

① Trapezoidal

$$\int f(x) = \frac{h}{2} [y_0 + y_n + 2(y_1 + y_2 + \dots + y_{n-1})]$$

② Simpson's $\frac{1}{3}$ rd (n = even)

$$\int f(x) = \frac{h}{3} [y_0 + y_n + 4(y_1 + y_3 + \dots) + 2(y_2 + y_4 + y_6 + \dots)]$$

③ Simpson's $\frac{3}{8}$ th

$$\int f(x) = \frac{3h}{8} [y_0 + y_n + 3(y_1 + y_2 + \dots) + 2(y_3 + y_4 + \dots)]$$

④ Eigen Values

$$\Rightarrow A \rightarrow |A - \lambda I| \rightarrow |A - \lambda I| = 0 \text{ solve for } \lambda.$$

These are eigen values.

$$\Rightarrow \text{Take } \lambda \text{ one by one} \rightarrow \text{put it into } (A - \lambda I) \begin{bmatrix} x \\ y \end{bmatrix} \rightarrow 0$$

convert to $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix}$ (a, b) are eigen vectors.

→ Eigen values of A^T matrix is just the diagonal elements.

→ Cayley Hamilton - every matrix satisfies its own characteristic eqn.

→ Hooke's law holds good upto proportional limit.

→ Coefficient of Restitution

$$e = 0.9 \Rightarrow v_{\text{after impact}} = 0.9 v_{\text{velocity before impact}}$$

$$\text{but } h_{\text{after impact}} \neq 0.9 h_{\text{before impact}}$$

→ $x = f(t)$

$$\frac{dx}{dt} = \text{instantaneous velocity}$$

$$\frac{d^2x}{dt^2} = \text{acc}^n$$

$$\text{average acc}^n = \frac{v(x_f) - v(x_i)}{t_f - t_i}$$

→ Velocity is tangent to streamline flow.

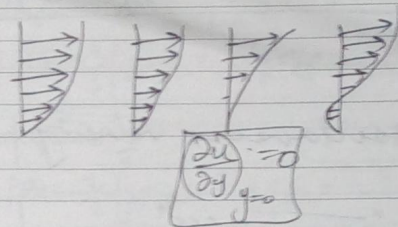
→ Laminar flow thro 2 concentric pipes

$$C = \left(\frac{\partial p}{\partial r} \right) \left(\frac{r}{2} \right)$$

$$u = \left(\frac{\partial p}{\partial r} \right) \left(\frac{R^2 - r^2}{4\mu} \right)$$

$$u = \left(\frac{\partial p}{\partial r} \right) \left(\frac{R^2}{8\mu} \right)$$

→ In a 2-D free stream flow, the separation point where fluid leaves the body in contact will be that point where the ~~the~~ fluid profile ~~separates~~ has zero slope on wall.



→ MKQ TGA Q35

→ $\nabla \times \vec{v} = 0 \Rightarrow$ irrotational

→ Buckling

→ both hinged $L = l_e$

→ one fixed other free $L_e = 2L$

→ one fixed other hinged $L_e = \sqrt{2}L$

→ both fixed $L_e = \frac{1}{2}L$

→ Boundary layer thickness = $\sqrt{\frac{Lx}{Re_x}} = \delta$

→ Q MKQ TGA Q46

$$E_x = \frac{1}{\epsilon} (\sigma_x - \mu \sigma_y)$$

$$E_y = \frac{1}{\epsilon} (\sigma_y - \mu \sigma_x)$$

$$v_{xy} = \frac{\sigma_{xy}}{\alpha}$$

$$E_{1,2} = \frac{E_x + E_y}{2} \pm \sqrt{\left(\frac{E_x - E_y}{2}\right)^2 + v_{xy}^2}$$

→ Thermal Engineering

$$Q = m C_p \Delta T$$

$$\boxed{dQ = m C_p dt} \quad \text{if } C_p = f(T)$$

$$dQ = m C_p dt = \boxed{m} \int f(T) dt$$

② Quasi-static process - locus of equlib point only

③ $Q_{a to b} = U_b - U_a + W_{a to b}$
depends only on final & initial pts

④ Gas constant $R = 287 \text{ J/kgK}$

⑤ $W < 0 \Rightarrow$ work is done on the system.

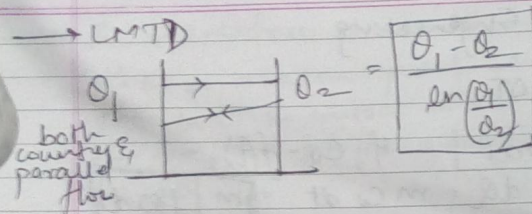
⑥ Kelvin-Planck - impossible to construct engine that takes in heat from a reservoir and converts it entirely to work.

⑦ Unavailable energy = $\Delta S x T_{\text{temp final}}$

⑧ Thermodynamic Relations

Maxwell's T, P, S, V \rightarrow $\frac{\partial h}{\partial p}_T = -\mu_J$ $\mu = \left(\frac{\partial T}{\partial p}\right)_h$

→ LMTD



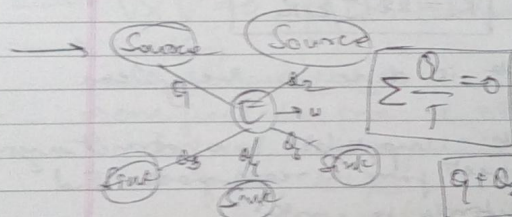
→ Spherical vessels

$$\sigma_c = \frac{Pd}{4t}$$

$$\rightarrow E = 2c \left(1 + \frac{1}{m} \right) \quad \left[\frac{1}{E} = \frac{1}{3c} + \frac{1}{9k} \right]$$

$$\boxed{\frac{1}{E} = \frac{1}{3C} + \frac{1}{9K}}$$

$$E = 3K \left(1 - \frac{2}{m} \right)$$



$$Q + Q_2 = W + Q_3 + Q_4 + Q_5$$

→ Partial pressure of a gas = $\frac{n_x V P_x}{V}$

$n = \text{no. of moles}$

p_v = vapour pressure

$$\sum n_i \psi_i$$

→ When talking about compression ratio, we refer to P_2/P_1 & not P_1/P_2 .

→ Water tube boilers are used for high pressure high output applications.

→ 1st law is valid for all processes, not just reversible or cycle.

→ For liquids and solids, $C_p = C_v$

→ Max. work that can be extracted

$$W_{\max} = (u_2 - u_1) - T_0 (s_1 - s_2) + p_0 (v_1 - v_2)$$

P_0, T_0 are different from $0, 0$.

$$U_2 - U_1 = mC_v \Delta T$$

→ Available Energy = $(P_1 - P_2) \times \frac{1}{\eta} \times \frac{1}{\gamma} \times \frac{1}{\beta}$
= $0.6 - 0.05$

→ Fluids

① For a flow to be physically possible,

$$\frac{\partial u}{\partial z} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0.$$

② ϕ - vel. potential

→ if it exists, there should be rotational

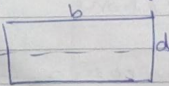
→ if ϕ satisfies Laplace eqn, it is irrotational flow

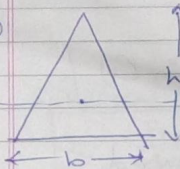
92 rotational flow

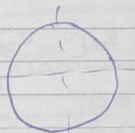
$$\left(\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0 \right)$$

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0$$

→ Moment of Inertia

①  $\frac{bd^3}{12}$

②  $\frac{bh^3}{36}$

③  $\frac{\pi d^4}{64}$

→ Gruebler's criterion

$$F = 3(n-1) - 2P_1 - P_2 - 6P_6 - 5P_5 - 4P_4 - 3P_3$$

P_i = no. of pairs with i dof

n = no. of links

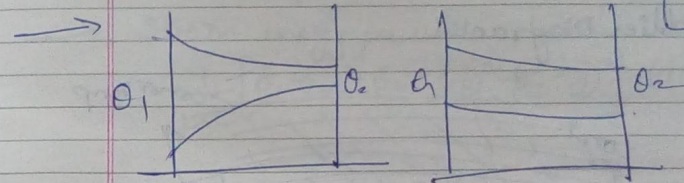
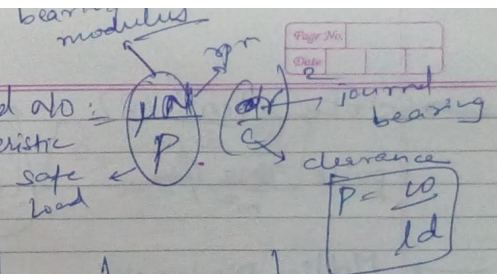
also, all the idle pairs are subtracted.

→ Bezier curve defined using n discrete points passes through 2 points.

→ Pressure $P = \frac{2}{3} E \rightarrow$ mean KE of gas

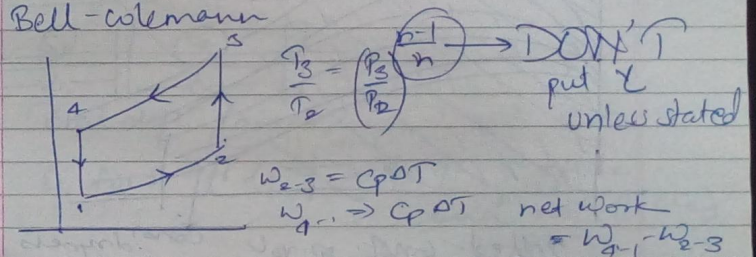
→ Availability = $\Delta Q - T \Delta S$
 $= \Delta Q - T \left[\sum \frac{Q_i}{T_i} \right]$

→ Sommerfeld no. = bearing characteristic number

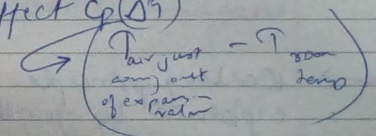


→ Pelton wheel \Rightarrow for max $\eta \rightarrow \frac{\text{jet vel}}{2} = \text{vel. of wheel}$

→ Bell-Colemann



net refrigerating effect $C_p(\Delta T)$



→ Differential Eqn
 $M dy + N dx = 0$
 for exact DE \rightarrow

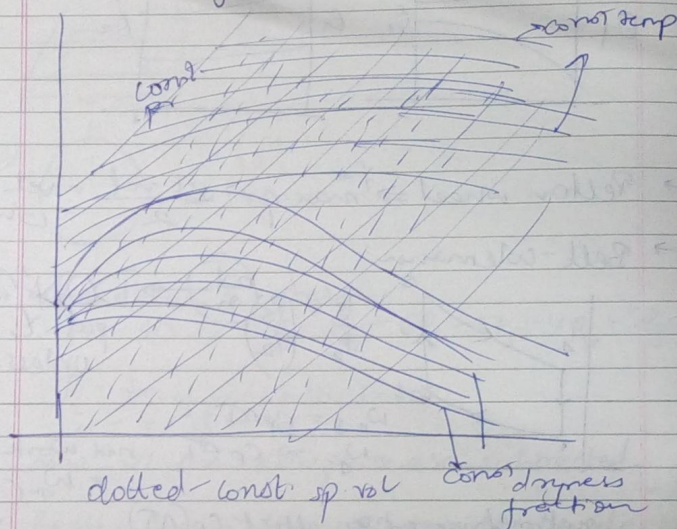
$$\frac{\partial M}{\partial x} = \frac{\partial N}{\partial y}$$

→ 1 stoke = $\frac{1 \text{ cm}^2}{\text{s}}$ (v)

1 poise = $\frac{\text{dynes}}{\text{cm}^2}$ (μ)

$$\eta = \frac{\mu}{\rho}$$

→ Mollier Diagram



→ Working principle of thermometer is best explained by zeroth law

→ Heating

$$\alpha \Delta T = \text{strain} = \frac{\Delta L}{L}$$

$$\alpha \Delta T \neq \Delta L$$

→ Shells

$$\sigma_1 = \sigma_2 = \frac{Pd}{4t}$$

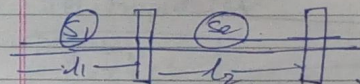
$$\sigma_2 = \sigma_3 = \frac{Pd}{4t}$$

→ Strain energy per unit volume

$$= \frac{1}{12C} [\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 + (\sigma_1 - \sigma_2)^2] \text{ (TGM)}$$

$$= \frac{1}{2C} [\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)] \text{ (Raj)}$$

→ Shafts in Series



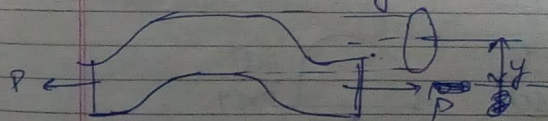
Total defn (angle) -

θ_1 for shaft l_1 + θ_2 for shaft l_2
and not $(l_1 + l_2)$

→ For shafts in torsion, θ in radians.

$$\frac{\tau}{R} = \frac{C\theta}{L} = \frac{T}{J_p}$$

→ Eccentric loading



$$\sigma = \frac{P}{A} + \frac{My}{I}$$

$$M = Py$$

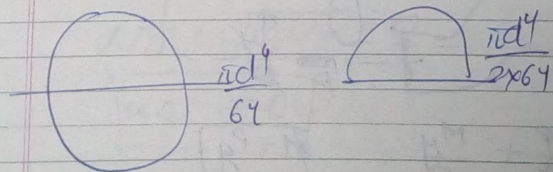
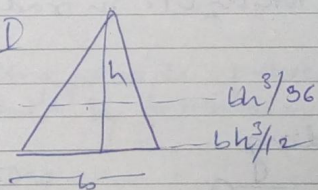
$$\begin{aligned} e_1 &= \frac{\sigma_1}{E} - \mu \frac{\sigma_2}{E} - \mu \frac{\sigma_3}{E} \\ e_2 &= \frac{\sigma_2}{E} - \mu \frac{\sigma_1}{E} - \mu \frac{\sigma_3}{E} \\ e_3 &= \dots \end{aligned}$$

$$\begin{aligned} \sigma_1 &= \frac{E}{1-\mu^2} (e_1 + \mu e_2) \\ \sigma_2 &= \frac{E}{1-\mu^2} (e_2 + \mu e_1) \\ \tau_{xy} &= \frac{E \mu}{2(1+\mu)} \end{aligned}$$

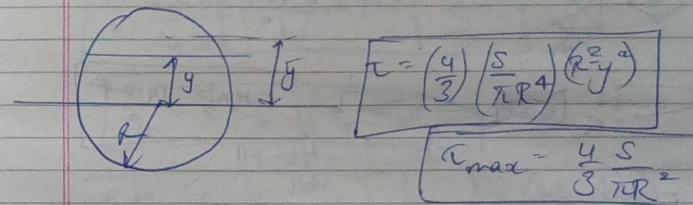
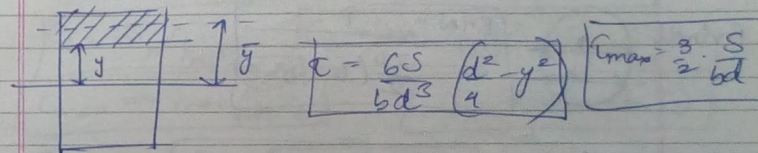
→ 2D stress system

$$\begin{bmatrix} \tau & -s \\ -s & c \end{bmatrix} = \begin{bmatrix} \sigma_n \\ c \end{bmatrix} \quad \text{and also} \quad \sigma_n = \sqrt{\sigma_n^2 + \tau^2}$$

→ MOD



→ Shear stresses



→ Jet Propulsion

Diffuser $T_2 = T_1 + \frac{C_a^2}{2g\eta_d}$

Nozzle exit vel = $\sqrt{2 \times \eta_n \times (T_1 - T_2)}$

Thrust $T = (G - C_a) \dot{m}$
 $\left(1 + \frac{m_f}{m_a}\right) (G - C_a) \dot{m}$

Thrust Power = $(G - C_a) C_a \dot{m}$

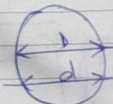
Propulsive power = $\frac{G \left(1 + \frac{m_f}{m_a}\right) G^2 - C_a^2}{2} = \frac{G^2 - C_a^2}{2}$

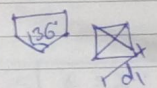
Propulsive efficiency = $\frac{2}{G/C_a + 1}$

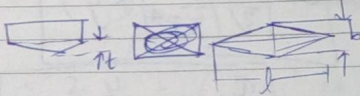
Thermal efficiency = $\frac{\left(1 + \frac{m_f}{m_a}\right) G^2 - C_a^2}{\left(\frac{m_f}{m_a} \times C_v\right)}$

Overall $\eta = \eta_{th} \times \eta_{prop}$

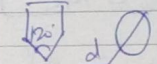
→ Hardness

Brinell 
$$BHN = \frac{P}{\frac{\pi D}{2} (D - \sqrt{D^2 - d^2})}$$

Vickers 
$$VHN = \frac{1.72 P}{d^2}$$

Rockwell 
$$KHN = \frac{14.2 P}{l^2}$$

 $l/b = 7/11$

Rockwell 

→ Composites

$x_1\%$ of E_1 & $x_2\%$ of E_2

$$E_{net} = \frac{x_1 E_1}{100} + \frac{x_2 E_2}{100}$$

→ Hydraulic Radius = $\frac{4 \text{ Area}}{\text{Perimeter}}$

→ Heat Exchangers

$$\epsilon = \frac{Q_{actual}}{Q_{maximum}}$$

$$C_h = \dot{m}_h C_{p_h}$$

$$C_c = \dot{m}_c C_{p_c}$$

$$C_{max} = \max(C_h, C_c)$$

$$C_{min} = \min(C_h, C_c)$$

$$R = \frac{C_{min}}{C_{max}}$$

$$NTU = \frac{UA}{C_{min}}$$

$$Q = \epsilon C_{min} (t_{h1} - t_{c1})$$

$$\epsilon = \frac{C_c (t_{c2} - t_{c1})}{C_{min} (t_{h1} - t_{c1})} = \frac{C_h (t_{h1} - t_{h2})}{C_{max} (t_{h1} - t_{c1})}$$

Parallel $\epsilon = \frac{1 - e^{-NTU(1+R)}}{1+R}$

Counter flow $\epsilon = \frac{1 - e^{-NTU(1-R)}}{1 - R e^{-NTU(1-R)}}$

→ Psychrometry

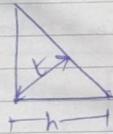
absolute humidity $w = \frac{\text{mass of water vapour}}{\text{mass of dry air}} = \frac{0.622 P_v}{P_a} = \frac{0.622 P_v}{P_t - P_v}$

degree of saturation $\mu = \frac{\text{mass of dry vapour in unit mass of dry air}}{\text{mass of vapour associated with saturated unit mass of dry air}} = \frac{P_v}{P_{vs}} \left(\frac{P_t - P_{vs}}{P_t - P_v} \right)$

relative humidity $\phi = \frac{1.6 w P_a}{P_{vs}}$

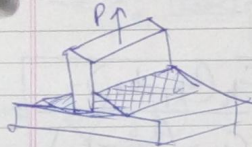
→ Welding

Parallel Fillet Weld



$$P = t l c = \frac{h l c}{\sqrt{2}}$$

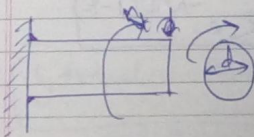
Transverse Fillet Weld



$$P = t l c = \frac{h l c}{\sqrt{2}}$$

$$P_{\max \text{ parallel}} = \frac{h l c}{\sqrt{2}} = 0.707 h l c$$

$$P_{\max \text{ transverse}} = 0.828 h l c$$



$$\tau = \frac{M_t}{2 \pi r^2}$$

$$\tau = \sqrt{\left(\frac{Q b}{2}\right)^2 + \tau^2} = \sqrt{\left(\frac{P \times x}{\pi r^2}\right)^2 + \left(\frac{P}{\pi r t}\right)^2}$$

→ Whirling of shafts

for whirling,

$$\left(\frac{m \omega^2}{EI}\right)^{1/4} = \frac{\pi}{l}, \frac{2\pi}{l}, \frac{3\pi}{l} \dots$$

$$\Rightarrow \omega^2 = \frac{\pi}{l} \left(\frac{EI}{m}\right)^{1/4}, \frac{2\pi}{l} \left(\frac{EI}{m}\right)^{1/4}, \frac{3\pi}{l} \left(\frac{EI}{m}\right)^{1/4} \dots$$

$$f_n = (\pi \omega) = \frac{\pi}{2} \sqrt{\frac{EI}{ml^4}}, \frac{4\pi}{2} \sqrt{\frac{EI}{ml^4}}, \frac{9\pi}{2} \sqrt{\frac{EI}{ml^4}} \dots$$

$$\text{or } f_n = \frac{n\pi}{2} \sqrt{\frac{EI}{ml^4}} \rightarrow \text{critical speed of shaft for } n \text{ nodes}$$

critical speeds

$$\text{lowest critical speed} = 3562 \sqrt{\frac{gEI}{wl^4}}$$

$$\text{higher} = \frac{\pi(n+1/2)}{2} \sqrt{\frac{gEI}{wl^4}} \quad \text{lowest almost corresponds to } n=1$$

$$\begin{aligned} \text{Tangential acc}^n &= 2N\omega \\ \text{Angular acc}^n &= \omega^2 r \\ \text{Resultant} &= \sqrt{a_t^2 + a_n^2} \end{aligned}$$

→ Linear eqns are of form

$$P_1 \frac{d^4 y}{dx^4} + P_2 \frac{d^3 y}{dx^3} + P_3 \frac{d^2 y}{dx^2} + P_4 \frac{dy}{dx} + P_5 = X$$

→ Available Energy

$$= \underset{\substack{\downarrow \\ \text{GOT}}}{Oh} - \underset{\downarrow}{TOS} + \underset{\substack{\downarrow \\ \text{KE}}}{\left(\frac{V^2}{2}\right)} + \underset{\substack{\downarrow \\ \text{PE}}}{\left(\frac{gz}{2}\right)}$$

$\begin{matrix} \text{as per} \\ \text{P.T.} \\ \rightarrow \text{C.R.} \\ \rightarrow \text{C.C.} \\ \rightarrow \text{C.R.} \end{matrix}$

→ Load fluctuating from $x \text{ kN}$ to $y \text{ kN}$,

$$\sigma_{\text{mean}} = \frac{x+y}{2} \quad \sigma_{\text{amplitude}} = \frac{x-y}{2}$$

→ Electrochemical machining

$$\text{mass removed per unit time} = \frac{m}{t} = \frac{EI}{F \rho}$$

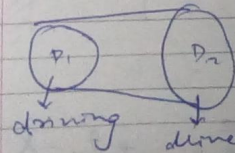
E = electrochemical equivalent = $\frac{\text{atomic weight}}{\text{valency}}$

I = current

F = Faraday's constant = 96500

ρ = density

→ Belt Drive



$$\text{Velocity Ratio} = \frac{N_2}{N_1} = \frac{D_1 + t}{D_2 + t}$$

peripheral speed of driving pulley
 $= \omega_1 \left(\frac{D_1 + t}{2} \right)$

S_1 = slip b/w belt & driving pulley
 S_2 = $\frac{\text{driven pulley}}{\text{driving pulley}}$

$$\text{speed of belt on driving pulley} = \omega_1 \left(\frac{D_1 + t}{2} \right) \left(\frac{100 - S_1}{100} \right)$$

this is also the speed of belt on driven pulley ~~also~~

$$\text{Speed of driven pulley} = \left[\omega_1 \left(\frac{D_1 + t}{2} \right) \left(\frac{100 - S_1}{100} \right) \left(\frac{100 - S_2}{100} \right) \right]$$

$$\text{length of belt - open belt} = \pi(R+r) + \frac{(R-r)^2}{4C} + 2C$$

$$\text{crossed belt} = \pi(R+r) + \frac{(R+r)^2}{C} + 2C$$

Tensions

$$\frac{T_1}{T_2} = e^{\mu \theta} \quad T_1 = \text{tight side Flat belt}$$

$$\frac{T_1}{T_2} = e^{\frac{\mu \theta}{\sin \alpha}} \quad \text{V-belt}$$

$$\text{Power transmitted} = (T_1 - T_2) V$$

$$\text{centrifugal tension} = m V^2 = T_c$$

mass per unit length

$$\text{max power transmitted when } T_c = \frac{T}{3} \quad V_{\text{max}} = \sqrt{\frac{T}{3m}}$$

→ Vector Calculus

Green's Theorem

$$\int_C (P dx + Q dy) = \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dx dy$$

Stokes Theorem

$$\int_C \mathbf{F} \cdot d\mathbf{R} = \int_S \text{curl } \mathbf{F} \cdot \mathbf{N} ds$$

Gauss Divergence

$$\int_S \mathbf{F} \cdot \mathbf{N} ds = \int_V \text{div } \mathbf{F} dv$$

→ A specimen is loaded in the plastic region and then unloaded.

- linear elastic region is increased
- ductility is reduced
- yield strength is increased

$$\rightarrow \text{Transmissibility} = \frac{\sqrt{1 + (2g)^2}}{\sqrt{(1-r^2)^2 + (2g)^2}}$$

→ length of a curve between x_1 & x_2

$$\int_{x_1}^{x_2} \sqrt{1 + \left(\frac{dy}{dx} \right)^2} dx$$

→ If a cycle is anti clockwise, work done is positive, else negative.

$$\rightarrow \epsilon_x = \frac{\sigma_x}{E} - \frac{\nu}{mE} (\sigma_y + \sigma_z)$$

$$\epsilon_y = \frac{\sigma_y}{E} - \frac{\nu}{mE} (\sigma_x + \sigma_z)$$

$$\epsilon_z = \frac{\sigma_z}{E} - \frac{\nu}{mE} (\sigma_x + \sigma_y)$$

$$\sigma_x = \frac{E}{(1+m)(1-2m)} [(1-m)\epsilon_x + m(\epsilon_y + \epsilon_z)]$$

$$\sigma_y = \frac{E}{(1+m)(1-2m)} [(1-m)\epsilon_y + m(\epsilon_x + \epsilon_z)]$$

$$\sigma_z = \frac{E}{(1+m)(1-2m)} [(1-m)\epsilon_z + m(\epsilon_x + \epsilon_y)]$$

$$\begin{aligned} \text{strain energy} &= \frac{1}{2} (\sigma_x \epsilon_x + \sigma_y \epsilon_y + \sigma_z \epsilon_z) \times V \\ &= \frac{1}{2} \frac{E}{(1+m)(1-2m)} [(1-m)(\epsilon_x^2 + \epsilon_y^2 + \epsilon_z^2) + 2m(\epsilon_x \epsilon_y + \epsilon_y \epsilon_z + \epsilon_z \epsilon_x)] \end{aligned}$$

$$\rightarrow T ds = du + p dv$$

$$\text{Availability} = \Delta U - T ds + p \Delta V$$

→ For a given system of eqns.

$\begin{bmatrix} - & - & - \\ - & - & - \\ 0 & 0 & 1 \end{bmatrix}$, if we get $\begin{bmatrix} - & - & - \\ - & - & - \\ 0 & 0 & 1 \end{bmatrix}$, the system has ∞ solns.

→ When one stress is larger than other two, the point is under uniaxial loading and all failure theories are applicable.

→ Radial accⁿ = $\omega^2 r$
Tangential accⁿ = $\omega^2 r$

→ Kaplan $R = \frac{\pi}{4} (D^2 - d^2) \sqrt{\frac{K}{\rho}} \sqrt{\frac{L}{g}}$
 $U = \frac{K}{\rho} \sqrt{\frac{L}{g}}$
speed ratio \downarrow flow rate

→ MRR in ascending order LBM, FBM, USM, EDM, ECM

→ Coefficient of volume expansion = $\frac{1}{V} \frac{dV}{dT}$

→ Flywheel

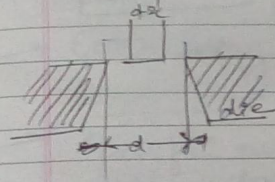
P = power

work done per cycle = $\frac{P}{N/60}$

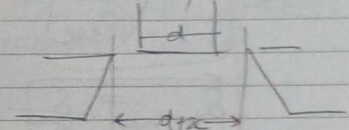
max fluctuation of energy = $\frac{P}{N/60} \times C_e$ (coeff of fluct energy)

$$\Delta E = U_0 = P w^2 C_s$$

→ ~~Sheet~~ Blanking
cutout piece is desired



Piercing
cut out part is waste



→ Maximum clearance = UL of hole - UL of shaft
~~minimum~~ allowance = UL of hole - UL of shaft
(with sign)

→ Extrusion

K = extrusion ratio = $\frac{A_0}{A_f}$
 A_f → dia of extruded part

strain = $\ln(K)$
~~deformation~~ $\ln(K)$

extrusion ram pressure = $\sigma \ln(K)$

→ In stress-strain diagram, if an object is loaded ~~with~~ beyond its plastic limit, and then unloaded, then when it is loaded again, ~~the new plastic~~ ^{proportional} limit now will be till the previous stress point.

→ Structure

$$m = 2j - 3$$

no. of members

no. of joints

imperfect
perfect
redundant

→ Boundary layer

$$\frac{\delta}{x} = \frac{5}{\sqrt{Re_x}}$$

$$C_{f_x} = \frac{0.664}{\sqrt{Re_x}}$$

$$T_f = \frac{1.328}{\sqrt{Re}}$$

$$\delta_{thermal} = \frac{\delta}{\sqrt{Pr}}$$

Laminar

$$h_c = 0.332 \left(\frac{k}{x} \right) \sqrt{Re} \sqrt{Pr} \quad \bar{h} = 0.664 \left(\frac{k}{L} \right) \sqrt{Re} \sqrt{Pr}$$

$$Nu_x = 0.332 \sqrt{Re} \sqrt{Pr} \quad Nu = 0.664 \sqrt{Re} \sqrt{Pr}$$

Turbulent

$$h_x = 0.0288 \left(\frac{k}{x} \right) Re_x^{1/4} Pr^{1/3} \quad \bar{h} = 0.036 \left(\frac{k}{L} \right) Re^{1/4} Pr^{1/3}$$

$$Nu_x = 0.0288 Re_x^{1/4} Pr^{1/3} \quad Nu = 0.036 Re^{1/4} Pr^{1/3}$$

$$\frac{\delta}{x} = \frac{0.371}{\sqrt{Re}}$$

$$\frac{u}{U} = \left(\frac{y}{\delta} \right)^{1/2}$$

→ Poisson's ratio = $\frac{\text{Lateral strain}}{\text{Linear strain}}$

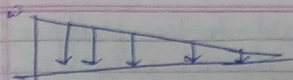
→ moment area method

$$\theta_{max} = \frac{A}{EI}$$

A of BMD

\bar{x} = distance of CG from left end

$$\Delta_{max} = \frac{Ax}{EI}$$

→  $y = \frac{wk^2}{20EI}$

→ strain energy caused by bending = $\int \frac{M^2 dx}{2EI}$

→ Euler's theory is NOT for eccentric loads.

→ Specific modulus / specific stiffness = $\frac{E}{\rho}$

→ yield stress - max resistance to deformation per unit area

proof stress - allowable resistance to deformation per unit area

→ Stress Ratio - ratio of min stress to max stress.

→ $\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \dots$

→ Cauchy's Theorem

$$f(a) = \frac{1}{2\pi i} \int \frac{f(z) dz}{(z-a)}$$

$$f^n(a) = \frac{n!}{2\pi i} \int \frac{f(z) dz}{(z-a)^{n+1}}$$

Residue

$$\text{Res } f(a) = \lim_{z \rightarrow a} [(z-a) f(z)]$$

$$\text{Res } f(a) = \frac{1}{(n-1)!} \left[\frac{d^{n-1}}{dz^{n-1}} (z-a)^n f(z) \right]$$

→ Velocity profile is said to be fully developed when it doesn't vary along the length of pipe.

→ uniform pressure theory $R = \frac{2}{3} \left(\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2} \right)$
(new clutch)

→ Bearing

loss of power = $P = T \times \omega$



$$= (F) \times r \times \omega =$$

$$= \left[\begin{matrix} \text{shear} \\ \text{stress} \times \text{area} \end{matrix} \right] \times r \omega$$

$$= (a \times 2\pi r) \times r \omega$$

$$= \left(\frac{\mu v}{c} \times 2\pi r l \right) \times r \omega = \left(\frac{\mu v}{c} \times 2\pi r l \right) \times r \omega$$

$$= \left(\frac{\mu v \omega}{c} \times 2\pi r l \right) \times r \omega$$

$$= \left[\frac{2\pi \mu r^3 l \omega^2}{c} \right]$$

→ Mechanical Advantage = $\frac{\text{output effort}}{\text{input effort}}$

→ Turbine

input pressure, vel given $E_1 = \frac{P}{\rho g} + \frac{v^2}{2g}$

output pressure, vel given $E_2 = \frac{P}{\rho g} + \frac{v^2}{2g}$

head loss = $E_1 - E_2 = h_L$

Power produced = $\rho g Q h_L$

$$dQ = m C_p dT$$

$$dU = m C_v dT$$

$$L_0 = \left(\frac{C}{P} \right)^{\frac{1}{P-1}} \left(\frac{P-1}{3} \right)^{\frac{1}{P-1}} L_{10} = \frac{60 \times 10^6}{10^6}$$

$$\left(\frac{L}{L_0} \right) = \left[\frac{\ln(r)}{\ln(r_{g0})} \right]^{1/1.7}$$

→ TRUSS

① Method of joints

→ Find reaction at base joints.

→ For a given joint, ~~find~~ resolve forces along horizontal or vertical dirⁿ to find remaining forces

② method of sections

→ Find reaction at joints

→ Cut one section passing through some members. Select one side (left, say). Pick a point on right.

For all the forces in left section, find their moments about a point on right & equate to zero

→ Soderberg

$$\frac{\sigma_m}{S_{yt}} + \frac{\sigma_a}{S_e} = \frac{1}{f_s}$$

Goodman

$$\frac{\sigma_m}{S_{ut}} + \frac{\sigma_a}{S_e} = \frac{1}{f_s}$$

Ceslob

$$\left(\frac{\sigma_m}{S_{ut}}\right)^2 f_s + \frac{\sigma_a}{S_e} = \frac{1}{f_s}$$

Recrystallization temp
 $\frac{2}{3} \rightarrow \frac{1}{2}$ of melting point

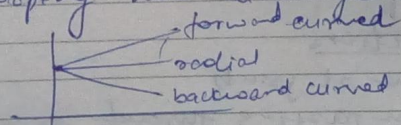
→ Ideal gas laws valid for gases low pressure and high temp.

→ For real skew symmetric matrix, eigen values are purely imaginary.

→ Coriolis component is present in shaper.

→ Eigen values of symmetric stress matrix are diagonal elements

→ Impeller with backward curved blades gives a drooping head discharge characteristic.



$$\frac{X_n}{X_{n-1}} = e^{g \Delta t}$$

→ Uniform distribution

$$\text{mean} = \frac{a+b}{2}$$

$$\text{variance} = \frac{(b-a)^2}{12}$$

→ Euler's theorem for homogeneous function

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = n \frac{F(u)}{F'(u)}$$

$$\text{Velocity} = \sqrt{V_r^2 + V_t^2} = \sqrt{V_r^2 + (\omega r)^2}$$

→ When gears rotate, cone power transmitted, torque may vary.

→ Slope of vaporisation curve is always positive

$$E(x) = \int_{-\infty}^{\infty} x f(x) dx \quad E(x^2) = \int_{-\infty}^{\infty} x^2 f(x) dx$$

$$\text{Var}(x) = E(x^2) - [E(x)]^2$$

$$\text{var}(ax+b) = a^2 \text{var}(x)$$

- When 2 gears mesh,
 ① gear is subjected to axial, shear & bending stress
 ② gear shaft is subjected to horizontal & vertical bending moment and twisting

→ Shrinkage of weld joint

transverse unrestrained joint = $1.52 \times \frac{t}{l} \times \frac{t}{l}$

length t
 $\frac{t}{l}$
 l
 plate thickness

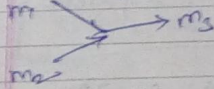
→ clearance per side = $0.0032 \sqrt{R}$
 for punching.

Die dia = $D + 2 \times C$

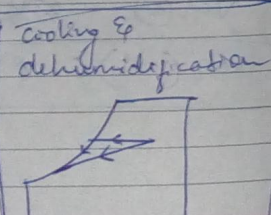
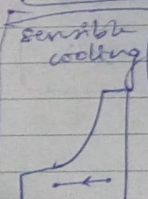
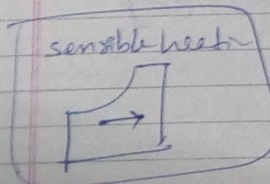
→ Displacement thickness = $\delta^* = \frac{\delta}{3}$
 momentum thickness = $\theta = \frac{\delta}{8}$

→ clearance for shearing = $0.0032 \sqrt{R}$

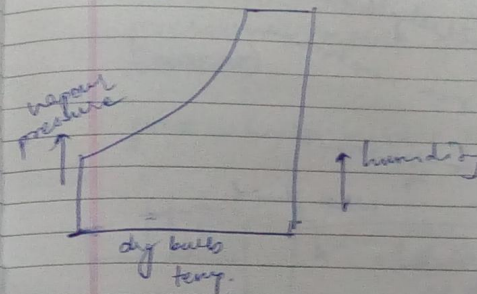
→ mixing of air streams



$$\frac{m_1}{m_2} = \frac{w_2 - w_3}{w_1 - w_3} = \frac{h_3 - h_2}{h_1 - h_3}$$



→ Psychrometric chart



$$P_v = P_{vs} - \frac{[P_t - (P_{vs})_{wb}][t_{db} - t_{wb}]}{1527.4 - 1.3 t_{wb}}$$

$$L[f(t)] = \int_0^{\infty} e^{-st} f(t) dt$$

→ Press forging - applying a continuous pressure on faces.

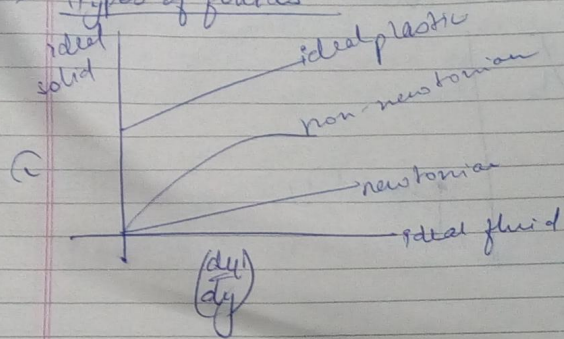
Upset forging - increases diameter of work piece by compressing its length. Usually done with high speed machines using crank press.

→ Homogeneous function

$\phi(x, y) \rightarrow x^n \phi(t/x) \rightarrow$ homogeneous of degree n .

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = nu$$

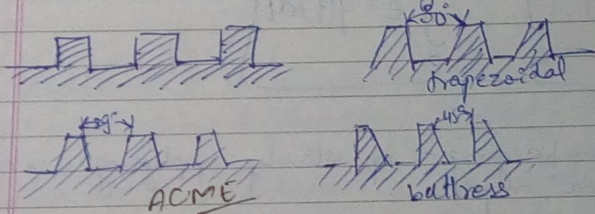
Types of fluids



Bolt Convention

M5 — nominal diameter = 5 mm

ISO metric — thread angle = 60°



M6x1 → nominal die 6 mm pitch = 1 mm

Froude's Number

$$Fr = \frac{v}{\sqrt{gl}}$$

→ velocity of flow
→ length of pipe

Capillarity

pressure inside a ^{solid} bubble $p = \frac{4\sigma}{d}$

for a hollow bubble $p = \frac{8\sigma}{d}$ → surface tension

$$h = \frac{4\sigma \cos \theta}{\rho g d} \approx \frac{4\sigma}{\rho g d}$$

for $\theta = 128^\circ$ $\theta_{H_2O} \approx 0$

Matrix

symmetric $A = A^T$

skew-symmetric $A = -A^T$

idempotent $A^2 = A$

adjoint (A) = (matrix of co-factors)^T of matrix A

$$A^{-1} = \frac{\text{adj } A}{|A|}$$

- Double-overhung turbine develops power P .
power for each turbine = $P/2$

$$N_S = \frac{N \sqrt{P}}{H^{5/4}} \rightarrow \text{power in kW}$$

→ Brayton cycle

$$\gamma_{p_{\max}} = \left(\frac{T_3}{T_1} \right)^{\frac{\gamma}{\gamma-1}}$$

$$v = C \sqrt{2gh} \quad u = \phi \sqrt{2gh} = \frac{\pi D \omega}{60}$$

- Drawing - length of job increased by decreasing its c/s area

- Upsetting - length of metal is reduced while increasing the cross-sectional area

- Swaging - hammering the metal shape while it is held on anvil with any one concave tools called swages.

- Coining closed die forging

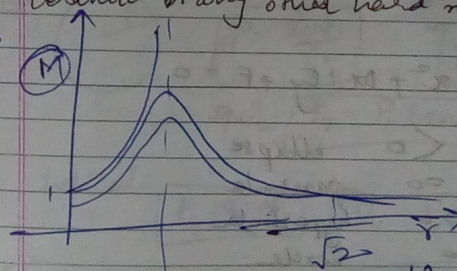
- Fullering Reduce dia & make it longer

- Moody's formulae

$$\left(\frac{1 - \eta_{\text{prototype}}}{1 - \eta_{\text{model}}} \right) = \left(\frac{D_m}{D_p} \right)^2 \left(\frac{H_m}{H_p} \right)^3$$

$$\eta = 1 - \frac{T_{\min}}{T_{\max}} \gamma_p^{\frac{\gamma-1}{\gamma}} \quad \text{optimum } \gamma_p = \frac{T_3}{T_1}^{\frac{\gamma}{\gamma-1}}$$

- Shot peening - producing compressive residual stress by bombarding metal with small spheres of ceramic or any other hard material. Prevents fatigue & stress corrosion failures



$\frac{W}{W_n}$ reduces after $\sqrt{2}$.

→ Radiation shield

without shield

$$Q_{net} = \frac{A_1 \sigma (T_1^4 - T_2^4)}{\left(\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1 \right)}$$

with shield

$$Q = \frac{A_1 \sigma (T_1^4 - T_2^4)}{\left(\frac{1}{\epsilon_1} + \frac{1}{\epsilon_3} - 1 \right) + \left(\frac{1}{\epsilon_2} + \frac{1}{\epsilon_3} - 1 \right)}$$

$$\epsilon_1 = \epsilon_2 = \epsilon_3 \quad T_3^4 = \frac{T_1^4 + T_2^4}{2}$$

n shields

$$Q = \frac{A_1 \sigma (T_1^4 - T_2^4)}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_n} + 2 \sum \frac{1}{\epsilon_i} - (n+1)}$$

→ Intensive prop → independent of mass content
point - pr temp
path - heat flow

→ Conic

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

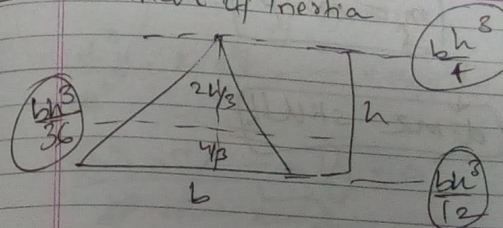
$$B^2 - 4AC < 0 \quad \text{ellipse}$$

$$B^2 - 4AC = 0 \quad \text{parabola}$$

$$B^2 - 4AC > 0 \quad \text{hyperbola}$$

$$A=C, B=0 \quad \text{circle}$$

→ Moment of Inertia



→ Section modulus

$$\frac{\sigma}{y} = \frac{M}{I} \Rightarrow \frac{I}{(y)} = \frac{M}{\sigma} \rightarrow \text{section modulus}$$

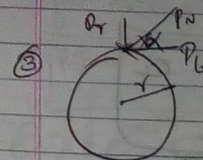
→ Polar modulus = $\frac{I_p}{R}$ I_p = polar moment of inertia

→ Gears

① Pressure angle (α)

② no. of teeth to avoid interference =

$$Z = \frac{2}{\sin^2 \alpha}$$



$$P = \frac{2\pi NT}{60}$$

$$P_t \times r = T$$

spur $P_r = P_t \tan \alpha$

helical

$$P_t \times r = M_t$$

$$P_r = P_t \frac{\tan \phi}{\cos \psi}$$

ϕ = helix angle

$$\cos \psi = \frac{\tan \alpha}{\tan \phi}$$

$$d = \frac{m_z}{\cos \psi}$$

$$m = m \cos \psi$$

$$d = m_z \text{ still!!!}$$

→ Nozzles

$$\text{Exit Velocity} = \sqrt{4.72 \sqrt{h_1}}$$

$$\text{Friction factor} = 44.72 \sqrt{K_1}$$

$$C = \sqrt{2 \left(\frac{n}{n+1} \right) P_1 V_1 \left[1 - \left(\frac{P_2}{P_1} \right)^{\frac{n+1}{n}} \right]}$$

$$m = \frac{AC}{V_2}$$

$$\text{critical pressure ratio} = \left(\frac{2}{n+1} \right)^{\frac{n}{n+1}}$$

$$P_1 V_1 \rightarrow P_2 V_2$$

→ SFEE

$$u_1 + \frac{P}{\rho g} + \frac{V^2}{2g} + Z + pV + Q = C$$

\downarrow internal energy
 \downarrow height
 \downarrow energy need for loss of fluid
 \downarrow heat supplied

$$Dh = m c_p \Delta T$$

$$Du = m c_u \Delta T$$

→ Complimentary Function

$$\frac{d^n y}{dx^n} + K_1 \frac{d^{n-1} y}{dx^{n-1}} + \dots + K_n y = 0$$

$$(D^n + D^{n-1} + \dots + 1)y = 0$$

solve the eqn.

$$(A) (D-m_1)(D-m_2) \dots (D-m_n) = 0$$

$$y = C_1 e^{m_1 x} + C_2 e^{m_2 x} + \dots$$

$$(B) m_1 = m_2$$

$$y = (C_1 + C_2 x) e^{mx}$$

$$(C) m_{1,2} = \alpha \pm i\beta$$

$$y = C_1 e^{(\alpha + i\beta)x} + C_2 e^{(\alpha - i\beta)x} + \dots$$

→ Particular Integral for the given differential eqn

$$PI = \left(\frac{1}{D^n + D^{n-1} + \dots + K_n} \right) X$$

$$(A) X = e^{ax}$$

$$\frac{D^n e^{ax}}{D^n e^{ax}} = \frac{a^n e^{ax}}{a^n e^{ax}}$$

$$\frac{1}{f(D)} e^{ax} = \frac{1}{f(a)} e^{ax}$$

$$(B) X = \sin(ax+b) \text{ or } \cos(ax+b)$$

$$\frac{1}{f(D)} \sin(ax+b) = \frac{1}{f(a^2)} \sin(ax+b) \quad \text{if } f(a^2) \neq 0$$

$$\text{doe } \frac{1}{f(D)} \sin(ax+b) = \frac{x}{f'(a^2)} \sin(ax+b) \quad \text{if } f'(a^2) \neq 0$$

and so on. Add powers of x as well.

$$d = \frac{m_n z}{\cos \psi}$$

$$m_n = m \cos \psi$$

$$d = m z \text{ still!!}$$

→ Nozzles

$$\text{Exit Velocity} = 44.72 \sqrt{h_1}$$

$$\text{Friction factor} = 44.72 \sqrt{k h_1}$$

$$C = \sqrt{2 \left(\frac{n}{n+1} \right) P_1 V_1 \left[1 - \left(\frac{P_2}{P_1} \right)^{\frac{n+1}{n}} \right]}$$

$$P_1 \rightarrow P_2 V_2$$

$$m = \frac{AC}{V_2}$$

$$\text{critical pressure ratio} = \left(\frac{2}{n+1} \right)^{\frac{n}{n+1}}$$

→ SFEE

$$u_1 + \frac{P}{\rho g} + \frac{V^2}{2g} + z + pV + Q = C$$

\downarrow internal energy \downarrow height \downarrow energy need for flow of fluid \downarrow heat supplied

$$\Delta h = m c_p \Delta T$$

$$\Delta u = m a \Delta T$$

→ Complimentary Function

$$\frac{d^n y}{dx^n} + K_1 \frac{d^{n-1} y}{dx^{n-1}} + \dots + K_n y = 0$$

$$(D^n + D^{n-1} + \dots + K_n) y = 0$$

solve the eqn.

$$(A) (D - m_1)(D - m_2) \dots (D - m_n) = 0$$

$$y = C_1 e^{m_1 x} + C_2 e^{m_2 x} + \dots$$

$$(B) m_1 = m_2$$

$$y = (C_1 + C_2 x) e^{m x}$$

$$(C) m_{1,2} = \alpha \pm i\beta$$

$$y = C_1 e^{(\alpha + i\beta)x} + C_2 e^{(\alpha - i\beta)x}$$

→ Particular Integral for the given differential eqn

$$PI = \left(\frac{1}{D^n + D^{n-1} + \dots + K_n} \right) X$$

$$(A) X = e^{ax}$$

$$D^n e^{ax} = a^n e^{ax}$$

$$\frac{1}{f(D)} e^{ax} = \frac{1}{f(a)} e^{ax}$$

$$(B) X = \sin(ax+b) \text{ or } \cos(ax+b)$$

$$\frac{1}{f(D)} \sin(ax+b) = \frac{1}{f(a^2)} \sin(ax+b) \quad \text{if } f(-a^2) \neq 0$$

$$\text{else } \frac{1}{f(D)} \sin(ax+b) = x \frac{1}{f'(a^2)} \sin(ax+b) \quad \text{if } f'(-a^2) \neq 0$$

and so on. Add powers of x as well.

④ $X = x^m$

$$\frac{1}{f(\omega)} x^m = [f(\omega)]^{-1} x^m$$

expand $f(\omega)$ using binomial theorem and solve

⑤ $X = e^{ax} V$

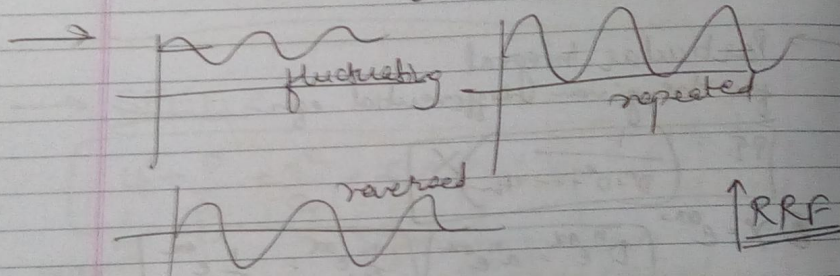
$$\frac{1}{f(\omega)} \left(\frac{dV}{dx} \right) = \frac{e^{ax}}{f(\omega)} \frac{1}{V} V$$

$V = f(\omega)$

$$\frac{1}{f(\omega) x^m} e^{ax} = \frac{x^m}{m!} e^{ax}$$

→ Rubbing velocity
= (relative angular velocity) \times (radius)

→ Entire function -
fn that is analytic in a range



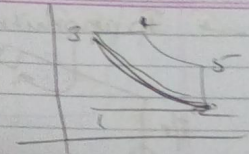
→ critical speed = $\sqrt{g/\phi}$
critical speed is independent of eccentricity

→ diesel engine

compression ratio = V_3/V_2

expansion ratio = V_5/V_4

cut-off ratio = V_4/V_3



→ Powder metallurgy - Fe/Al/Cu

→ $Q = AV$

Q = vol. flow rate

m = mass flow rate = ρQ

→ Auto-collimator - straightness

→ interferometry - flatness

→ Hobbing is the most accurate method for making gears
or maybe gear shaping with pinion cutter

→ decrement ratio

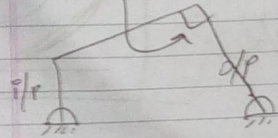
is not same as damping ratio

$$\text{decrement ratio} = \frac{x_n}{x_m} = e^{\frac{\pi \zeta}{\sqrt{1-\zeta^2}}}$$

damping ratio = ζ

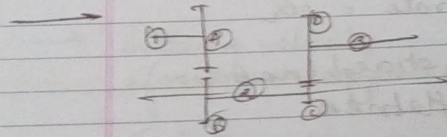
$\frac{2}{3} \pi \times \frac{1}{3} \times \frac{1}{3}$

→ Transmission angle for 4 bar



→ Von mises theory of failure

$$\sigma_e = \sqrt{\sigma_x^2 + \sigma_y^2 + \sigma_z^2 - \sigma_x \sigma_y}$$



net radial force acting on shaft ^{sum of} = radial forces of in gear B + C

→ Vibrations

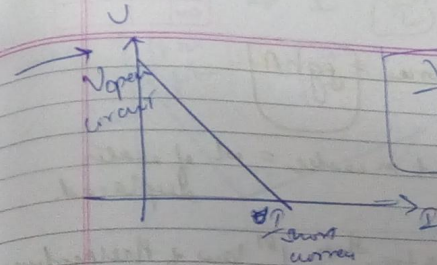
n^{th} critical speed $\propto n^2$

$$n^{\text{th}} \rightarrow \text{critical speed} = \frac{\text{frequency}}{(\text{no. of nodes})^2} \times h^2$$

→ Torque required to lift a load

$$T = \frac{F \cdot d_m}{2} \left(\frac{1 + \pi \mu d_m}{\pi d_m - \mu l} \right) + \frac{F \mu c d_c}{2}$$

(A-P/S) np → notch collar dia



$$\frac{V}{V_{oc}} + \frac{P}{P_{sc}}$$

$$L[f^n(x)] = s^n F(s) - s^{n-1} f(0) - s^{n-2} f'(0) \dots - f^{(n-1)}(0)$$

$$W_{\text{pump}} = VAP$$

→ DPT - temperature at which vapour condenses into liquid

→ Critical point - point at which no phase boundaries exist.

Usually known as the vapour-liquid critical point, above which, distinct liquid and gas phases did not exist.

Triples Point is that point where all three states exist at the same time.

critical point specifies the condition at which there ceases to be a boundary between the phases.

TP - all three states ~~has~~ are in equilibrium

CP - liquid and gas have the same density (vapour-liquid CP)

→ Buoyancy force = $\rho g h A$

wt in air - wt in water = wt of water displaced

→ PMM ① - violate the 1st law of thermodynamics that would produce useful energy without any energy source or produce more energy than consumed.

PMM ② - violate 2nd law of thermodynamics produce useful energy from a single reservoir in equilibrium only

→ Properties of green sand -

- ① Porosity / Permeability $\frac{50-26}{P\%} \rightarrow \text{mm}$
- ② Plasticity
- ③ Adhesiveness
- ④ Cohesiveness
- ⑤ Refractoriness

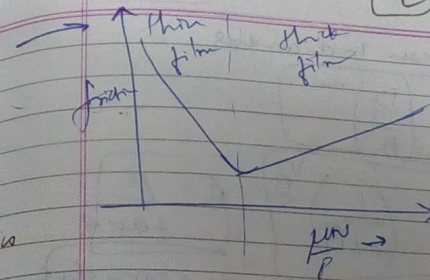
→ Auto-collimator - (straightness) small angles with high sensitivity.

→ $V(x) = x$
 $V(ax) = a^2 V(x)$

→ $P(A|B) = \frac{P(A \cap B)}{P(B)}$

$P(A \cap B) = P(\overline{A \cup B}) = 1 - P(A \cup B)$

$\frac{C}{a} - \frac{r}{L} = \frac{e\phi}{I}$ $\left(\frac{C\phi}{L}\right)$



Friction circle radius = $r \times \mu$

→ If $K_m = 0$, it means that material is rigid \times
 " perfectly plastic \times
 there is no longitudinal strain in the material \times

→ Stress Concentration (K_t)
 notch sensitivity $q = \frac{K_t - 1}{K_t}$ actual stress concentration factor

$S_e = K_a K_b K_c K_d S_u$
 limit \downarrow size \downarrow reliability $\rightarrow \frac{1}{K_f}$ endurance limit
 \uparrow theoretical stress concentration factor

→ if $|A| = 0$ eigen values of inverse don't exist

→ sagging hogging

→ $0.45 \sqrt{D} + 0.001 D$

→ Maximum Reduction In diameter

$$R = 1 - \left(\frac{D_f}{D_i} \right)^2$$

$$R = 1 - \left(\frac{1}{1+B} \right)^{1/3}$$

$$B = \frac{\mu}{\tan \alpha}$$

α = semi die angle

→ Strain energy = $\frac{P\delta}{2}$

→ max η of power transmission through pipes is $\frac{2}{3}$

→ $HP = 745 \cdot 6W$

→ Grinding

Manufacturers Code / Abrasive Chemical / Grain size / Grade / Structure / Bond / Suffix

→ Another expression $dP = Mdx + Ndy$ would be a thermodynamic property iff

$$\left(\frac{\partial M}{\partial y} \right)_x = \left(\frac{\partial N}{\partial x} \right)_y$$

Chip Thickness Ratio = $\frac{\text{uncut chip thickness}}{\text{cut chip thickness}}$

→ Energy losses in Pipes

① Enlargement = $\frac{V_1 - V_2}{2g}$

② contraction = $\frac{0.5V^2}{2g}$

③ entrance = $\frac{0.5V^2}{2g}$

④ exit = $\frac{V^2}{2g}$

⑤ gradual contraction / expansion = $\frac{K(V_1 - V_2)^2}{2g}$

⑥ bends = $\frac{KV^2}{2g}$

⑦ fittings = $\frac{KV^2}{2g}$

fractional loss
= $\frac{4fLV^2}{gD}$

Hydraulic Grade line (HGL)

when a liquid flows through pipes encountering several losses in the way, and if we try to plot the pressure head at various places, we will get a straight line sloping downwards throughout.

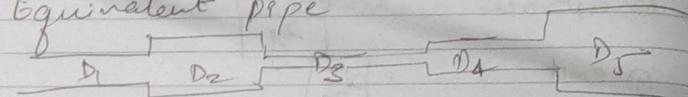
Total energy line (TEL)

line connecting the energy points from input to output.

Pipes In Series

Head losses are added linearly.
 $H = \sum \frac{4fLV^2}{2gD}$ (say, ignoring all others).

Equivalent pipe



$$\frac{L}{D^5} = \sum_{i=1}^5 \frac{L_i}{D_i^5}$$

Pipes In Parallel

$$Q = Q_1 + Q_2$$

Head loss is same $h_f = \frac{4fLQ_1^2}{2gD_1^5} = \frac{4fLQ_2^2}{2gD_2^5}$

Governor

Heat Conduction in Sphere

$$\frac{t_1 - t_2}{\frac{1}{k} \ln \frac{r_2}{r_1}} = \frac{Q}{4\pi k r_1 r_2}$$

$$Q = \frac{4\pi k r_1 r_2 (t_1 - t_2)}{\ln \frac{r_2}{r_1}}$$

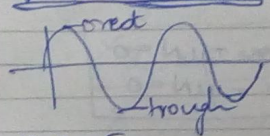
Cylinders

$$Q = \frac{4\pi k L (t_1 - t_2)}{\ln \frac{r_2}{r_1}}$$

$$\frac{t_1 - t_2}{\frac{1}{k} \ln \frac{r_2}{r_1}} = \frac{Q}{4\pi k L}$$

$$A_m = \sqrt{A_1 A_2}$$

Vibration -



S-DOF

$$m\ddot{x} + kx = 0$$

$$\omega_n = \sqrt{\frac{k}{m}}$$

$$x(t) = A \cos \omega_n t + B \sin \omega_n t = A \cos(\omega_n t - \phi)$$

S-DOF damped (viscous)

$$m\ddot{x} + c\dot{x} + kx = 0 \Rightarrow m\lambda^2 + c\lambda + k = 0$$

$$x = G e^{\lambda_1 t} + H e^{\lambda_2 t}$$

$$\lambda_{1,2} = \frac{-c \pm \sqrt{c^2 - 4mk}}{2m} = \frac{-c}{2m} \pm \sqrt{\frac{c^2}{4m^2} - \frac{4k}{m}}$$

$c^2 > 4mk \rightarrow$ no vibration

$c^2 = 4mk \rightarrow$ no vibration

$c^2 < 4mk \rightarrow$ vibration

boundary condition $c^2 = 4mk \Rightarrow c_c = 2\sqrt{mk}$

Damping Ratio $\xi = \frac{c}{c_c}$

$$x(t) = e^{-\xi \omega_n t} [\cos \omega_d t + \xi \sin \omega_d t]$$

$$\omega_d = \omega_n \sqrt{1 - \xi^2}$$

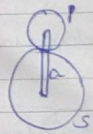
Energy loss per cycle \propto (Amp loss)²

$$\frac{x_n}{x_{n+1}} = e^{\frac{2\pi \xi}{\sqrt{1 - \xi^2}}} = e^{\frac{2\pi \xi}{\sqrt{1 - \xi^2}}}$$

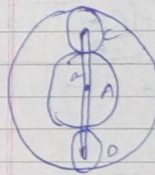
logarithmic decrement $\delta = \ln \left(\frac{x_n}{x_{n+1}} \right) = \frac{2\pi \xi}{\sqrt{1 - \xi^2}}$

$\approx 2\pi \xi$ ($\xi \leq 1$)

→ Epicyclic Gear Train



Rev(a)	Rev(s)	Rev (P)
0	1	$-\frac{r_s}{r_p}$
x	x	$-\frac{r_s}{r_p}x$
y	$y+x$	$y - x \frac{r_s}{r_p}$



$$\omega_B = \omega \left[\frac{r_A}{2} + r_c \right]$$

a	A	C or D	B
0	1	$-\frac{r_A}{r_c}$	$-\frac{r_A}{r_c} \times \frac{r_c}{r_B}$
0	x	$-\frac{x r_A}{r_c}$	$-\frac{x r_A}{r_c} \times \frac{r_c}{r_B}$
y	$y+x$	$y - \frac{x r_A}{r_c}$	$y - \frac{x r_A}{r_B}$

→ Collision of 2 bodies

→ momentum is conserved.

$$e = \frac{\text{velocity of separation}}{\text{velocity of approach}}$$

→ Isochronism of Governor

- governor with range of speed zero is isochronous.

→ Flywheel -

coefficient of fluctuation of speed = $\frac{\text{change in speed}}{\text{mean speed}}$

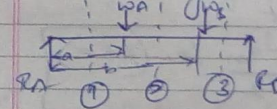
$$C_s = \frac{v_1 - v_2}{\frac{v_1 + v_2}{2}}$$

$$U_0 = I \omega^2 C_s$$

$$\text{coeff of steadiness} = \frac{1}{C_s}$$

$$\text{coeff of fluctuation of energy} = \frac{\text{max fluctuation of energy}}{\text{work done per cycle}}$$

→ Macaulay's Method

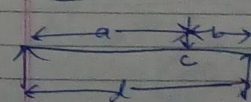


at ① $M = R_A x$

at ② $M = R_A x - w_A(x-a)$

at ③ $M = R_A x - w_A(a-a) - w_B(a-b)$

$$M = EI \frac{d^2 y}{dx^2}$$



$$y_c = \frac{w a b^2}{3 EI}$$

$$y_{\text{max}} = y \left(\sqrt{\frac{b^2 - a^2}{3}} \right) = \frac{w b (b^2 - a^2)}{9 \sqrt{3} EI}$$

→ Power Screws

$$d_{\text{mean}} = d_{\text{outer}} - 0.5p$$

Helix angle = α

$$\tan \alpha = \frac{l}{\pi d_m} \rightarrow \text{lead} \Rightarrow \text{dist the screws will move in one revolution}$$

$$\mu = \tan \phi$$

Torque Requirement

$$\text{lifting load} = M_t = \frac{W d_m \tan(\phi + \alpha)}{2}$$

$$\text{lowering load} = M_t = \frac{W d_m \tan(\phi - \alpha)}{2}$$

$$\eta = \frac{\tan \alpha}{\tan(\phi + \alpha)} \Rightarrow \eta_{\text{max}} = \frac{1 - \sin \phi}{1 + \sin \phi}$$

$$\eta_{\text{max}} \text{ occurs at } \alpha = 45^\circ - \frac{\phi}{2}$$

η_{max} depends only on μ .

→ Radiation

✓ non-black planes

Diagram showing two parallel plates with emissivities ϵ_1 and ϵ_2 . Arrows indicate radiation between them.

$$\text{Total energy that leaves surface ①} = Q_1 = \frac{\epsilon_1 \epsilon_2}{\epsilon_1 + \epsilon_2 - \epsilon_1 \epsilon_2}$$

Total energy that leaves surface ②

$$Q_2 = \frac{\epsilon_2 \epsilon_1}{\epsilon_1 + \epsilon_2 - \epsilon_1 \epsilon_2}$$

$$\text{Energy going from ① to ②} = Q_{12} = \frac{\sigma(T_1^4 - T_2^4)}{\left(\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1\right)}$$

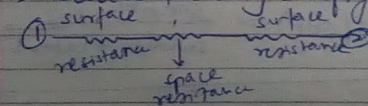
✓ non-black cylinders

$$Q_1 = \frac{\epsilon_1 \epsilon_2}{\frac{A_1 \epsilon_1 + A_2 \epsilon_2 - A_1 A_2 \epsilon_1 \epsilon_2}{A_2}}$$

$$Q_2 = \frac{\epsilon_2 \epsilon_1}{\frac{\epsilon_2 + \frac{A_2 \epsilon_1}{A_1} - \frac{A_1 \epsilon_1 \epsilon_2}{A_2}}{A_2}}$$

$$\text{Energy going from ① to ②} = Q_{12} = \frac{A_1 \sigma(T_1^4 - T_2^4)}{\frac{1}{\epsilon_1} + \frac{A_1}{A_2} \left(\frac{1}{\epsilon_2} + 1\right)}$$

✓ 2 bodies in any configuration



$$Q_{\text{net}} = \frac{E_{b1} - E_{b2}}{\left(\frac{1 - \epsilon_1}{A_1 \epsilon_1} + \frac{1}{A_1 F_{1-2}} + \frac{1 - \epsilon_2}{A_2 \epsilon_2}\right)}$$

→ Brake Power

$$\frac{(W - S) \pi (D + t) N}{60}$$

dead load spring balance dia of pulley rope thickness

→ For a Refrigerant R12 $\Rightarrow R_{xy} \Rightarrow y = \text{no. of fluorine molecules}$

$$h_f = \frac{fLV^2}{2gD}$$

pressure loss = egh_f

→ shafts

max principal stress theory

$$\sigma = \frac{16}{\pi d^3} \left[M_b + \sqrt{M_b^2 + M_t^2} \right]$$

max shear stress theory

$$\tau = \frac{16}{\pi d^3} \left[\sqrt{M_b^2 + M_t^2} \right]$$

→ Sensible Heat Factor (SHF) = $\frac{\text{sensible}}{\text{sensible} + \text{latent}}$

→ $\leftarrow C \rightarrow 0.8 \rightarrow C \leftarrow$
 Hypo-eutectoid Eutectoid Hyper-eutectoid

→ Molecular wt = $\sum \left(\frac{\% \text{ of a component}}{\text{Total \%}} \right) \times \text{wt of that component}$

→ Spectral Matrix is the one that consists of the eigen values in its principal diagonal.

→ interference $\text{path} = n\lambda$ → micrometer
 $2xy$ → micrometer

→ Pipe of Varying Diameter

$$\Delta P = \frac{128 \mu Q L}{\pi (D_1^4 - D_2^4)} \left[\frac{1}{D_1^3} - \frac{1}{D_2^3} \right]$$

→ Pump

$Q \propto N$ $HP \propto N^3$ $\left(\frac{H}{N^2 D^2} \right) = \text{const}$

→ Radiation emitted = $\sigma A T^4$
 $\sigma = 5.67 \times 10^{-8} \frac{\text{W}}{\text{m}^2 \text{K}}$

→ Extrusion force = $F = A_0 K \ln \left(\frac{A_0}{A_f} \right)$
 \downarrow
 initial area extrusion constant extruded part area

→ Poisson Distribution
very rare events but have a large no. of opportunities of occurrence

$$P(x) = \frac{m^x}{x!} e^{-m}$$

mean $m = np$
no. of trials n \downarrow prob. of success p

prob. of x successes

mean = m
SD = \sqrt{m}

$\mu_2 = m$

$\mu_3 = m + 3m^2$

skewness $\beta_1 = \frac{1}{m}$

kurtosis $\beta_2 = 3 + \frac{1}{m}$

→ Normal distribution

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$z = \frac{x-\mu}{\sigma}$$

→ Moment about \bar{x}

$$\mu_r = \frac{1}{N} \sum f_i (x_i - \bar{x})^r$$

skewness $\beta_1 = \sqrt{\beta_1} = \sqrt{\frac{\mu_3^2}{\mu_2^3}}$

kurtosis $\beta_2 = \frac{\mu_4^2}{\mu_2^2}$

→ Correlation

coeff of correlation $r = \frac{\sum xy}{n\sigma_x\sigma_y}$

$$x = x - \bar{x}$$

$$y = y - \bar{y}$$

σ_x = SD of x -series

σ_y = SD of y -series

W.R

regression coefficient

$$y - \bar{y} = r \frac{\sigma_y}{\sigma_x} (x - \bar{x})$$

$y - \bar{y} = r \frac{\sigma_y}{\sigma_x} (x - \bar{x})$

$$x - \bar{x} = r \frac{\sigma_x}{\sigma_y} (y - \bar{y})$$

$y - \bar{y} = r \frac{\sigma_y}{\sigma_x} (x - \bar{x})$

→ Continuous Probability distribution

cumulative Prob. Dist = $\int_{-\infty}^x f(x) dx$

Expectation $E(x) = \int_{-\infty}^{\infty} x f(x) dx$

$\int_{-\infty}^{\infty} x f(x) dx$

$\int_{-\infty}^{\infty} x f(x) dx$

$\int_{-\infty}^{\infty} x f(x) dx$

variance $\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x) dx$

Variance $V(x) = E(x^2) - (E(x))^2$

$E(x) = \sum x p_i$

$E(x^2) = \sum x^2 p_i$

$V(x) = E(x^2) - (E(x))^2$

→ Order of error in Simpson's rule is $\propto h^2$

→ Stalling of blades - air stream not able to follow the blade contour

→ Knocking

→ min speed for starting a centrifugal pump

$$N = 120 \eta \frac{V_{k2} D_2}{\pi (D_2^2 - D_1^2)}$$

→ Specific speed of a pump (Nk)

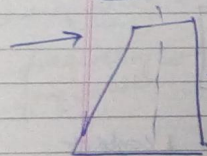
$$N_s = \frac{N \sqrt{Q}}{H_m^{3/4}}$$

$$\frac{N \sqrt{Q}}{H_m^{3/4}} = C$$

$$\frac{\sqrt{H_m}}{DN} = C$$

$$\frac{Q}{D^3 N} = C$$

$$\frac{P}{D^5 N^3} = C$$



Drop tube

→ apply Bernoulli's eqn on both sections

→ Pressure drop in venturi and orifice $\propto \left(\frac{\text{coeff of discharge}}{2} \right)^2$

→ Designation of Refrigeration

$$C_r H_s F_t C_g$$

$$S + t + y = 2r + 2$$

$$R_1 (r-1) (s+1) t$$

$$S + t + y = 2r$$

$$R_1 (r-1) (s+1) t$$

→ Nozzles

$$\text{exit velocity} = C = \sqrt{20h} = 44.72 \sqrt{K \frac{Oh}{L+H}}$$

friction factor

→ Knocking - / Detonation / Pinking - burning of fuel in one more points inside the cylinder apart from the usual shock wave set by the spark plug.

Prevention -

- ① fuel of high octane rating to be used
- ② increase air-fuel ratio
- ③ reduce load on engine

→ Pre-ignition

Ignition of fuel before sparking. Results from a hot-spot in the cylinder or too great a compression ratio for fuel.

→ Vibration
Reaction Turbine
Degree of Reaction

$$= \frac{\Delta h_{\text{rotor}}}{\Delta h_{\text{rotor}} + \Delta h_{\text{stator}}}$$

→ Centrifugal tension
 $T_{\text{light side}}$ for max power transmission

→ Isothermal $\rightarrow \int p dv = - \int v dp$

→ Critical thickness of insulation
 $= \frac{k}{h}$ or $\frac{2k}{h}$. This formula gives thickness
from center of pipe

→ Whirling of shafts
additional deflⁿ of rotor due to centrifugal force $y = \frac{e}{\left(\frac{\omega}{\omega_c}\right)^2 - 1}$
eccentricity of mass
frequency of vibration ω_c critical frequency

→ Gibbs Phase Rule

$$F + P - C = 2$$

degree of freedom \downarrow phase \downarrow no. of components

→ LMTD

when $\theta_1 = \theta_2$, $\text{LMTD} = \theta_1 = \theta_2 = \theta$ (itself)

→ 6X19 rope means
6 strands of 19 wires each

→ Elements Attributes
 \bar{x}, k p, n_p, c, μ

→ Best size wire $d = \frac{\text{Pitch} \cdot \sec(\frac{\theta}{2})}{2}$

→ Springs

$$\text{spring index} = \frac{D}{d} = \frac{\text{mean diameter}}{\text{dia. of wire}}$$

$$\text{pitch of coil} = \frac{\text{free length}}{n_t - 1}$$

$$C = \frac{8PD}{\pi d^3}$$

$$K = \frac{Gd^4}{8D^3N}$$

$$S = \frac{8PD^3N}{Gd^4}$$

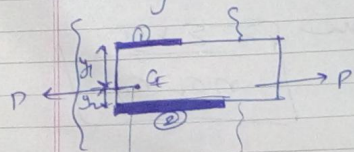
$$g = \frac{8PD^3N}{Gd^4}$$

$$\text{strain energy} = \frac{PS}{2}$$

→ n_1 & n_2 discs of multiple clutch, no. of contact pairs are $n_1 + n_2 - 1$.

→ Critical pressure^{ratio} for max discharge through a nozzle $= \left(\frac{2}{n+1}\right)^{\frac{n}{n-1}}$

→ Axially loaded Unsymmetrical welded Joints



CG of 2 welds
 $P_1 = \text{strength of ①}$
 $P_2 = \text{str. of ②}$

$$P_1 y_1 = P_2 y_2 \quad P_1 y_1 = t_1 y_1$$

$$t_1 y_1 = t_2 y_2 \quad P = P_1 + P_2$$

→ Eccentric loading in plane of weld

$$\left\{ \begin{array}{c} \text{eccentric loading} \end{array} \right\} = \left\{ \begin{array}{c} \text{primary} \end{array} \right\} + \left\{ \begin{array}{c} \text{secondary} \end{array} \right\}$$

$(M = P \times e)$

primary shear stress - due to force P

$$\sigma = P/A \quad A = A_1 + A_2 \text{ of both welds}$$

secondary shear stress

$$\tau_s = \frac{M r}{J}$$

$$J_{G1} = \frac{t l^3}{12} + \frac{A l^2}{12}$$

$$J_G = J_{G1} + A r_1^2$$

similarly find J_G for all welds.

$$\tau = \frac{M r}{J}$$

→ Thick shells

internal p

ext.

$$\sigma_r = -P_i D_i^2 \left[\frac{D_o^2}{4r^2} - 1 \right]$$

$$\sigma_r = \frac{-P_o D_o^2}{(D_o^2 - D_i^2)} \left[1 - \frac{D_i^2}{4r^2} \right]$$

$$\sigma_t = \frac{P_i D_i^2}{(D_o^2 - D_i^2)} \left[\frac{D_o^2}{4r^2} + 1 \right]$$

$$\sigma_t = \frac{-P_o D_o^2}{(D_o^2 - D_i^2)} \left[1 - \frac{D_i^2}{4r^2} \right]$$

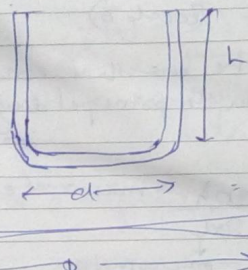
$$\sigma_c = -P_i D_i^2 \left[\frac{D_o^2 - D_i^2}{(D_o^2 - D_i^2)} \right]$$

$$t = \frac{D_i}{2} \left[\sqrt{\frac{\sigma_t + P_i}{\sigma_c + P_i}} - 1 \right] \text{ Lane's}$$

$$t = \frac{D_i}{2} \left[\sqrt{\frac{\sigma + (1-2\mu)P}{\sigma - (1+\mu)P}} - 1 \right] \text{ Clannino}$$

$$t = \frac{D_i}{2} \left[\sqrt{\frac{\sigma + (1-\mu)P}{\sigma - (1+\mu)P}} - 1 \right]$$

→ Blank size



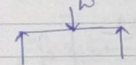
$$D = \sqrt{d^2 + 4dh} \quad \left(\frac{d}{r} \geq 20\right)$$

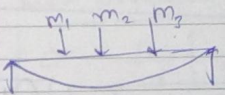
$$= \sqrt{d^2 + 4dh - 0.5r} \quad \left(15 \leq \frac{d}{r} \leq 20\right)$$

$$= \sqrt{d^2 + 4dh - r} \quad \left(10 \leq \frac{d}{r} \leq 15\right)$$

$$= \sqrt{(d-2r)^2 + 4d(h-r) + 8r(d-0.7r)} \quad \left(\frac{d}{r} \leq 10\right)$$

→ Vibrations

①  $f_n = \frac{1}{2\pi} \sqrt{\frac{g}{\Delta}}$ $\Delta = \frac{mg l^3}{3EI}$

②  $f_n = \frac{0.4985}{\sqrt{\Delta_1 + \Delta_2 + \dots + \Delta_5}}$ $\frac{1}{1.27}$

→ Average of a qty from 0 to x

$$= \frac{1}{x} \int_0^x f(x) dx$$

where $f(x)$ is local/instantaneous value

→ Stresses

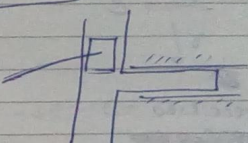
→ gradual load $\sigma = \frac{W}{A}$

→ suddenly applied load $= \frac{2W}{A} = \sigma$

→ impact load $\sigma = \frac{W}{A} \left[1 + \sqrt{1 + \frac{2hAE}{Wl}} \right]$

→ loss of head in "pipes"

$$\frac{\Delta p}{\rho g} = \frac{32 \mu u l}{\rho g D^2}$$

→ Scotch gate  generates sine function

→ Kaplan

$$Q = \frac{\pi (D_o^2 - D_b^2)}{4} \sqrt{h} \quad u_1 - u_2 = \frac{\pi D_o^2 n}{60}$$

$$\text{Area of flow} = \frac{\pi (D_o^2 - D_b^2)}{4}$$

→ Probability

Bayes's Thm $P(A/B) = \frac{P(A) P(B/A)}{\sum P(A) P(B/A)}$

$A/B = A$ such that B has already happened

Bernoulli Trials

$p =$ probability of success $q = 1 - p$

probability of r successes out of n trials $= {}^n C_r p^r q^{n-r}$

$\mu_2 = npq$ $\mu_3 = npq(p-q)$ $\mu_4 = npq[1 + 3(n-2)pq]$

$B_1 = \frac{\mu_2^2}{\mu_1^2}$ $B_2 = \frac{\mu_4}{\mu_2^2}$ $\text{skewness} = \frac{1-2p}{\sqrt{npq}}$ $\text{kurtosis} = B_2$

$$X = \frac{F_0/K}{\sqrt{(2gr)^2 + (1-r^2)^2}}$$

$$r_{\text{peak}} = \sqrt{1-2gg^2}$$

→ Viscous flow between 2 concentric pipes

$$v = \left(\frac{\partial p}{\partial r} \right) \left(\frac{r}{2} \right)$$

$$u = \left(\frac{\partial p}{\partial r} \right) \left(\frac{R^2 - r^2}{4\mu} \right)$$

$$\bar{u} = \left(\frac{\partial p}{\partial r} \right) \left(\frac{R^2}{8\mu} \right)$$

$$\Delta p = \frac{32\mu \bar{u} L}{D^3}$$

u_{max} occurs at $r=0$

$$\frac{u_{\text{max}}}{\bar{u}} = 2$$

Viscous flow between 2 parallel plates

$$u = \left(\frac{\partial p}{\partial x} \right) \left(\frac{y^2 - y^2/2}{2\mu} \right)$$

$$v = \left(\frac{\partial p}{\partial x} \right) \left(\frac{t - 2y}{2} \right)$$

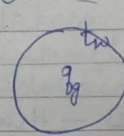
$$\bar{u} = \left(\frac{\partial p}{\partial x} \right) \left(\frac{t^2}{12\mu} \right)$$

$$\frac{\partial p}{\partial x} = \frac{2\tau}{y}$$

$$\frac{u_{\text{max}}}{\bar{u}} = \frac{3}{2}$$

$$\Delta p = \frac{12\mu Q L}{3\pi (D_1^3 - D_2^3)} \left[\frac{1}{D_1^3} - \frac{1}{D_2^3} \right]$$

→ Cylinder with heat generation



$$\frac{t - t_w}{t_{\text{max}} - t_w} = 1 - \left(\frac{r}{R} \right)^2$$

$$t = t_w + \frac{q_g (R^2 - r^2)}{4k}$$

Energy generated per unit time within the rod = energy dissipated per unit volume, at the surface

$$(q_g)(\pi R^2 L) = (h)(2\pi R L)(t_w - t_a)$$

$$t = t_a + \frac{q_g R}{2h} + \frac{q_g (R^2 - r^2)}{4k}$$

$$t = t_w + \frac{q_g (R^2 - r^2)}{4k}$$

→ Heat Generation within a slab
same temp on both sides

$$t = \frac{q_g (2L - x)}{2k} x + t_w$$

$$[t_{\text{max}} \text{ at } \frac{L}{2}]$$

different temp on both sides

$$t = \left[\frac{q_g (L-x)}{2k} + \frac{t_{w2} - t_{w1}}{L} \right] x + t_{w1}$$

→ Coriolis Component of Acceleration

$$= 2\omega V$$

→ Gyroscope

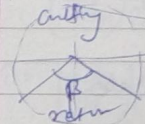
An object rotating about an axis — this axis rotating about another axis perpendicular to it — as a result, the entire setup will rotate about an axis \perp to both of them.

$$\alpha = \omega \omega_p \quad T = \frac{2\pi}{\alpha}$$

Plane viewed from backside —

Propellers rotating in clockwise sense, plane takes a left turn, its nose will go up and tail will go down.

→ Quick Return mechanism



$$\frac{\text{Time of cutting stroke}}{\text{Time of return stroke}} = \frac{360/\omega}{\beta}$$

$$\begin{bmatrix} \cos \theta & - \\ - & \sin \theta \end{bmatrix} \begin{bmatrix} \sin 2\theta \\ 1 \\ \cos 2\theta \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

→ No. of Instantaneous Centers

$$= N = \frac{n(n-1)}{2} \quad n = \text{no. of links}$$

Kennedy's Theorem — if three bodies have relative motion among themselves then 3 centers must lie on a straight line.

List ①

Brinell
Jickers
Rockwell
Scleroscope

List ②

Tungsten Carbide Ball
Diamond pyramid
Diamond cone
Diamond tipped hammer

→ Statistical Considerations In Design

$$\hat{\sigma} = \sqrt{\frac{\sum f_i x_i^2}{n} - \frac{(\sum f_i x_i)^2}{n^2}}$$

$$z = \frac{x - \mu}{\sigma}$$

→

Time taken to fill a mold

$$= \frac{\text{Volume}}{A_{\text{gate}} \sqrt{2gh}}$$

$$L[e^{at} f(t)] = f(s-a)$$

$$L[t^n f(t)] = (-1)^n \frac{d^n}{ds^n} [f(s)]$$

$$L[f(t)] = \int_0^\infty e^{-st} f(t) dt$$